# Analysis of Flight Computer Data From Off-Vertical Trajectories

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## INTRODUCTION

This paper was originally to be about off-vertical accelerometer analysis, or *OVAA*. In order to demonstrate that method, though, it became necessary to compare inertial altitudes with barometric altitudes. A number of legacy problems quickly emerged. The task of dealing with them fell upon me like the task of cleaning house after retrieving a fallen ring from a congregation of dust bunnies.

The paper therefore became elliptical, insofar as it acquired two focal points:

1) Off Vertical Accelerometer Analysis (OVAA); and
2) Inertial/Barometric Closure (IBC)

The first topic is a subset of the second, but it is noteworthy because it is new.


## ACKNOWLEDGEMENTS

I owe special thanks to Michael Peck and Peter Pfingsten of the *Skybusters* rocket club, for sending my recovered rocket from Ohio to Pittsburgh.

I owe thanks to Drake Damerau of NEPRA, who was kind enough to post my software for download.

I also owe thanks to Brian Cole, Paul Hopkins, Geoff Huber, and Cliff Sojourner who graciously allowed me to include data from their flights in this paper.

In addition, David Schultz and John DeMar gave me invaluable advice along the way.

Thanks, Guys!




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# Part I: Off-Vertical Accelerometer Analysis (OVAA)

**PRESENT SITUATION**

Rocket enthusiasts are well aware that single-axis accelerometers do not measure lateral motion. In addition, many harbor the erroneous opinion that accelerometers are sensitive to gravity, and that they are therefore also sensitive to the rocket's angle with respect to the horizontal. For these reasons, single-axis accelerometers are thought to be unsuitable for the analysis of off-vertical trajectories.

Even so, accelerometers are frequently used in off-vertical missions. Some launch angles are judged to be ever so close to vertical. Such angles are rarely measured, but if they were, a few would stray $10^0$ or more from the straight up. Sometimes the rocket tips off, encounters launcher whip, or weathercocks as it leaves the rod. All such difficulties are ignored in the hope that this time, vertical analysis will yield pleasing numbers: numbers that agree with the much more reliable barometric altimeter. Alas, they rarely do.

Ah, but three axis accelerometers are about to swing in on a rope and save the day – that is, as soon as each of us can afford one. This author is not saving his pennies, however, because the real problem isn't three-dimensional. Those three dimensions are pinned to the reference frame of a ground observer. The real observer, in this case, is the instrument aboard the rocket, and that instrument's frame is accelerated (or so we would hope). In order to convert observations from that frame to the ground-based observer's frame, we must account for six degrees of freedom: three translational and three angular.

Make a gun with your thumb and index finger. Now turn your wrist so that your thumb rotates part way around your finger. If your finger is the long axis of a rocket, and your thumb is an accelerometer on a perpendicular axis, there is no way to relate your thumb's readings to the outside world if you do not know its angle of rotation about your finger. That's why real rockets carry gyroscopes as well as accelerometers. At this writing, gyroscopes would appear to be some years away from the hobby rocket market.

**GRAVITATIONAL OBLIVION**

By the way, accelerometers are oblivious to gravity. OK, I lied. They can sense tidal forces, but tidal forces are miniscule on the earth's surface, so I didn't lie very much, now, did I?

The standard counterexample concerns a rocket sitting vertically on a horizontal surface as illustrated.



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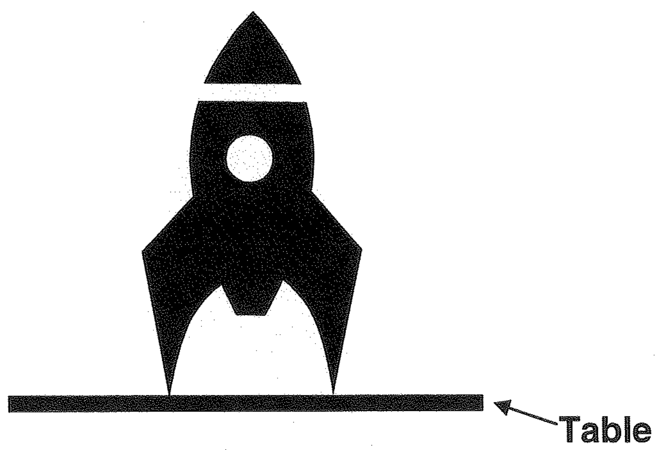
# On Earth
# Accelerometer Registers 1g
# Rocket is Motionless




An onboard accelerometer registers 1*g*. In fact, we would normally *calibrate* anything it registers as 1*g*, and the rocket itself is motionless. The reading is normally thought to be a reaction to gravity, so you'll pardon me if I point out that it is not. The reading is, in fact, a reaction to the force imposed by the table underneath, which is equal to the earthly weight of the rocket, and this would be a species of thrust.

If we imposed the same force in outer space (see illustration below), the rocket would accelerate at 1*g* and that is exactly what the accelerometer would read – just as it did with the same amount of thrust back on earth. Gravity did not affect the accelerometer reading, but it did affect the motion of the rocket.

We can even do the reverse experiment. In space, we need only cut the thrust. The rocket stops accelerating, and the instrument registers 0*g*. We get the same reading if we pull the table out from under our earthly rocket. That rocket then falls at a rate of -1*g*. Once again, <u>the accelerometer registers the same thing with and without gravity. It is the rocket itself that is affected by gravity</u>.

Gravitational oblivion is the rule that emerges. All accelerometers are oblivious to gravity, and that includes three axis accelerometers and six axis accelerometers. Because they do not sense gravity, all gravitational information must be supplied exogenously from theory. **Accelerometer analysis is therefore semi-empirical. Its empirical data must be supplemented with theoretical data. The gravitational influence must be simulated!**



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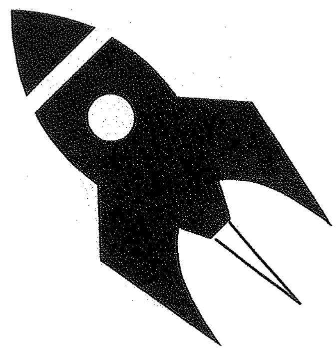
Free Space
Accelerometer Registers 1g
Rocket Accelerates at 1g



**<u>Thrust = Earthly Weight of Vehicle</u>**

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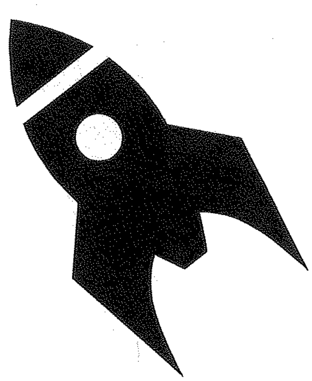
Free Space, No Thrust

Accelerometer Registers 0g





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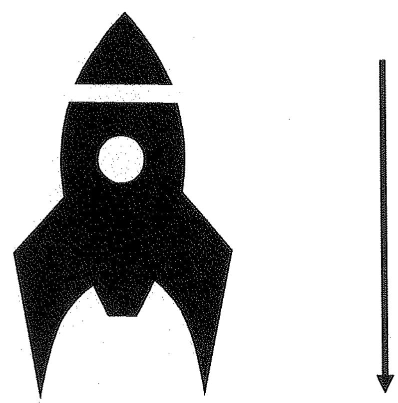
# Free Fall in Gravity

## Accelerometer Registers 0g




**OK, I LIED! (An exception to gravitational oblivion)**
As altitude increases, gravitational force and acceleration decrease. Consider the effect on an accelerometer, which is essentially a frame attached to a mass by means of a spring. If one orients such a device with the frame vertically above the mass and drops it, the gravitational acceleration on the frame is slightly less than it is on the mass. The mass should therefore pull the frame along and stretch the spring, rendering a nonzero reading. The force that causes this effect is called *Tidal Force*, and accelerometers are not oblivious to it; it's just that tidal force is infinitesimal near the earth's surface. If we were launching near a giant star or a black hole, we should have to expand our model to account for the effect. It isn't a factor in hobby rocketry.

**WHAT DO ACCELEROMETERS MEASURE?**
Here are the five major forces on a rocket:

1) Gravity (Can't detect it!)
2) Thrust
3) Drag
4) Lift
5) Buoyancy (Too small to worry about!)

We cannot detect gravity, and we won't bother ourselves with acceleration from buoyancy. A single-axis accelerometer, properly aligned, measures the vector components of the other three forces along the long axis of the rocket. The reading also



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has a component of unbiased noise. That is, if we were to average many noise values, we should get a number very close to zero.

why not italics?

$$Accelerometer\,\mathrm{Re}ading =$$
$$LongAxisThrustAcceleration + LongAxisDragAcceleration + LongAxisLiftAcceleration$$
$$+ UnbiasedNoise$$

A radially-symmetric rocket at zero angle of attack has no lift. Furthermore, the long axis vector component of the remaining forces is the whole of those forces. Thus,

**AT EXACTLY ZERO AOA:**

$$Accelerometer\,\mathrm{Re}ading = ThrustAcceleration + DragAcceleration + UnbiasedNoise$$

Rockets do not travel at exactly zero angle of attack. The formula changes somewhat at small angles.

**AT SMALL AOA:**

$$Accelerometer\,\mathrm{Re}ading = ThrustAcceleration + DragAcceleration +$$
$$BiasedError + UnbiasedNoise$$

Where UnbiasedError is very small at small angles of attack.

What?

At first glance, the last equation would seem a trivial modification of the previous equation. We apparently added a term, *BiasedError*, to suck up any error arising at small nonzero AOA. The actual picture is more subtle.

We are assuming that the small angles of attack come from small stability oscillations. Some aspects of these oscillations may have a positive or negative effect on the reading according to the angle. *UnbiasedNoise* increases a little. *DragAcceleration* also increases, because it is not zero-AOA drag. It is, however, more realistic value than zero-AOA drag.

Finally, there is a component to these oscillations that does not get small with averaging. With stability oscillations, the real trajectory has a 3D sinusoidal, lumpy character. The trajectory our analysis derives is smooth. That means the lumps have been pulled tight, and the derived trajectory is longer than the actual one. (Filtering may help this problem, but filtering is a big topic by itself.) The amount of this bias at any point is proportional
? to the inverse cosine of the angle of attack. Cosines of small angles are very close to unity. For example, the cosine of $2^0$ is 0.9994. The cosine of $15^0$ is only about 0.97, and that is quite a large deviation from zero AOA. The important point is that this term is small, and so

**UNDERLYING PRINCIPLE**

$$Accelerometer\,\mathrm{Re}ading \approx ThrustAcceleration + DragAcceleration$$

That, in a nutshell, is what a single axis accelerometer measures in a substantially ballistic trajectory.



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## VERTICAL ACCELEROMETER ANALYSIS PROGRAMS

Whether their authors realize it or not, all vertical accelerometer analysis programs reduce to vertical trajectory simulation programs that have been rewritten to take the sum of thrust and drag accelerometers directly from the accelerometer readings. Accelerometer analysis programs are actually simpler than vertical simulators, because a vertical simulator has to derive drag from velocity, altitude, and drag coefficient. Accelerometer analysis programs have the sum handed to them.

These programs simulate the effect of gravity on acceleration by assuming vertical orientation, and subtracting 1$g$ from the reading. Nothing in the data informs the programs that gravity is perfectly opposing the direction of motion. The programs assume that this is so, calculate what gravitational influence would be under that assumption, and adjust.

## VERTICAL FORMULAS

As noted, these are rewritten one-dimensional simulation formulas. (References 10, 12, and 13)

$$Acceleration_i = (\mathrm{Re}ading_i - g)$$

$$Velocity_i = Velocity_{i-1} + \frac{(\mathrm{Re}ading_i - g) + (\mathrm{Re}ading_{i-1} - g)}{2} * \Delta time$$

or

$$Velocity_i = Velocity_{i-1} + \left[\frac{\mathrm{Re}ading_i + \mathrm{Re}ading_{i-1}}{2} - g\right] * \Delta time$$

$$Altitude_i = Altitude_{i-1} + \frac{Velocity_i + Velocity_{i-1}}{2} * \Delta time$$

## OFF-VERTICAL ACCELEROMETER ANALYSIS PROGRAMS

The innovative point of departure for this paper is that <u>the underlying principle can just as easily serve as the basis for a two-dimensional simulation.</u> That is, we can rewrite a two-dimensional simulation to take the sum of thrust and drag accelerations directly from the accelerometer reading. In such a context, the reading is decomposed into its vertical and horizontal vector components. The effect of gravity is simulated by subtracting 1$g$ from the vertical vector component alone. Once the rocket leaves the launch rod, this procedure becomes a prescription for *Gravity Turning*, which tells us the angle, in the sky at any moment, of a stable rocket in a ballistic trajectory. <u>Before the rocket leaves the launch rod</u>, the procedure leads to equations that are just like the vertical equations with $g * \sin\theta$ substituted for $g$.

## OVAA LAUNCH ROD FORMULAS

That's right. In an off-vertical trajectory, we need to simulate flight along the launch rod, just as we do in a two-dimensional simulation. That means we have to measure, not only



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launch rod angle, but also launch rod length (preferably from its tip to the bottom of the launch lug). This is true because we have to add the influence of gravity, and gravity acts differently on the launch rod than it does in free flight. (References 10, 12, and 13)

$$Acceleration_i = \frac{(Reading_i + Reading_{i-1})}{2} - g * \sin\theta$$

$$Velocity_i = Velocity_{i-1} + \left[\frac{(Reading_i + Reading_{i-1})}{2} - g * \sin\theta\right] * \Delta time$$

$$Velocity_{yi} = Velocity_i * \sin\theta$$

$$Y_i = Y_{i-1} + \frac{Velocity_{yi-1} + Velocity_{yi}}{2} * \Delta time$$

$$Velocity_{xi} = Velocity_i * \cos\theta$$

$$X_i = X_{i-1} + \frac{Velocity_{xi-1} + Velocity_{xi}}{2} * \Delta time$$

Terminate these launch rod equations when

$$X_i^2 + Y_i^2 >= LaunchRodLength^2$$

**OVAA FREE FLIGHT FORMULAS**

Here are standard free-flight formulas, rewritten from standard two-dimensional simulations. (Reference 10, 12 and 13)

$$Acceleration_{yi} = -g + Reading_i \frac{Velocity_{yi}}{|Velocity_i|}$$

$$Acceleration_{xi} = Reading_i \frac{Velocity_{xi}}{|Velocity_i|}$$

$$|Velocity_i| =$$
$$\frac{Reading_i * \Delta time + \sqrt{(Acceleration_{yi-1} * \Delta time - g * \Delta time + 2Velocity_{yi-1})^2 + (Acceleration_{xi-1} * \Delta time + 2Velocity_{xi-1})^2}}{2}$$

$$Velocity_{yi} = \frac{Acceleration_{yi-1} * \Delta time - g * \Delta time + 2Velocity_{yi-1}}{\left[2 - \frac{Reading_i * \Delta time}{|Velocity_i|}\right]}$$



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$$Velocity_{xi} = \frac{Acceleration_{xi-1} * \Delta time + 2Velocity_{xi-1}}{\left[2 - \frac{Reading_i * \Delta time}{|Velocity_i|}\right]}$$

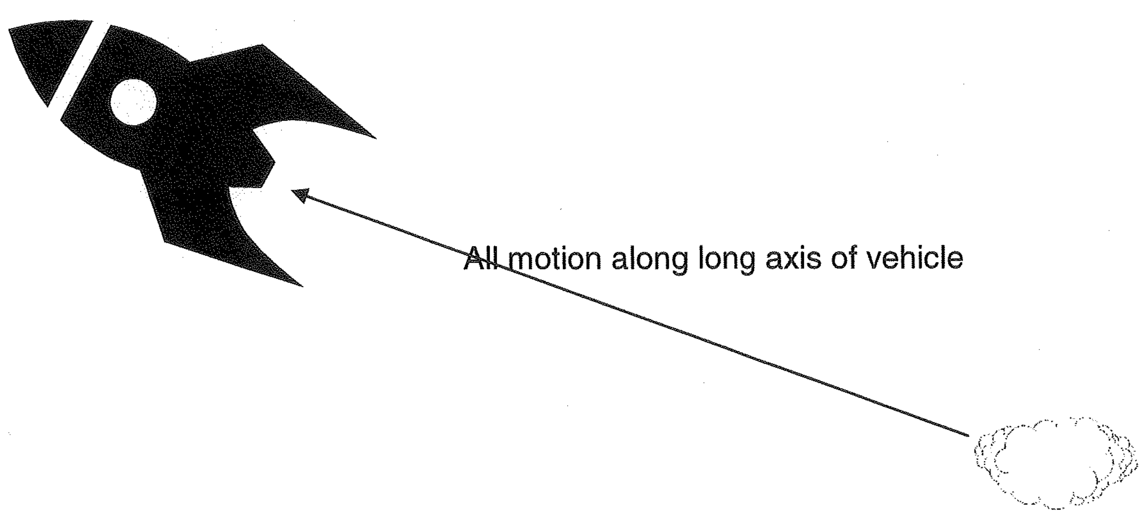
$$Y_i = Y_{i-1} + \frac{Velocity_{yi-1} + Velocity_{yi}}{2} * \Delta time$$

$$X_i = X_{i-1} + \frac{Velocity_{xi-1} + Velocity_{xi}}{2} * \Delta time$$

**WHAT HAPPENED TO LATERAL MOTION AGAIN????**

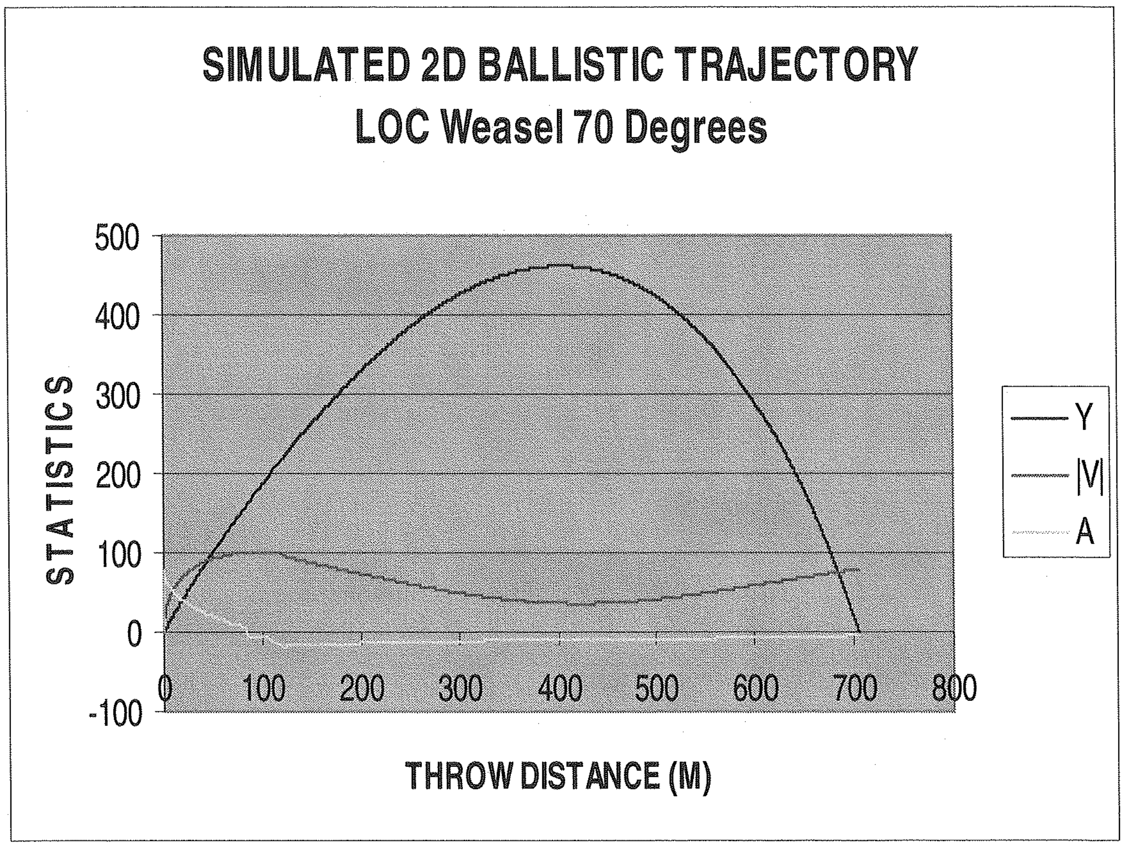
Consider a rocket following its nose in an oblique linear trajectory. The trajectory certainly has a horizontal component, and that is lateral motion to an observer on the ground. Thankfully, the ground observer's perspective is totally irrelevant, and accelerometers are perfectly capable of measuring horizontal motion. From the point of view of the rocket, there is no lateral motion. Indeed, all the motion is along the long axis of the vehicle!

**IN THIS <u>LINEAR TRAJECTORY</u>**
**there is a horizontal component**
**BUT NO LATERAL MOTION**
**from the rocket's perspective**



The next trick is to invoke a basic property of ballistic trajectories. **In an off-vertical ballistic trajectory, the only lateral influence is gravity!** Except during a perfectly horizontal moment, not all gravitational influence is lateral. In a vertical rise, none of the influence is lateral. It doesn't matter. **We cannot detect gravity at all, let alone its lateral component, but we are saved because its influence is brain dead simple to simulate!** Thus, in a **BALLISTIC** trajectory, we don't HAVE to sense lateral motion. We know the rocket's angle in the sky from the two-dimensional simulation, and we can simply INFER the lateral motion.

If all of those assumptions hold, a two-dimensional trajectory analysis program can output a trajectory much like one that is simulated from a thrust curve and drag coefficient, as illustrated in the graph below. This graph was output by the program 2DQD2.xls.




**THE FLY IN THE BUTTERMILK**

The problem with all this is that we cannot simply make ontological arguments and *define* trajectories to be ballistic. The trajectories, so-analyzed, have to actually *be* ballistic or everything falls through. Therefore, if the rocket takes a hard left turn in the middle of its flight (and haven't we all seen that happen?), we are out of luck. Significant lift undermines this model[1].

[1] Some lift is actually helpful to this kind of analysis – like the lift that keeps the rocket following its nose.



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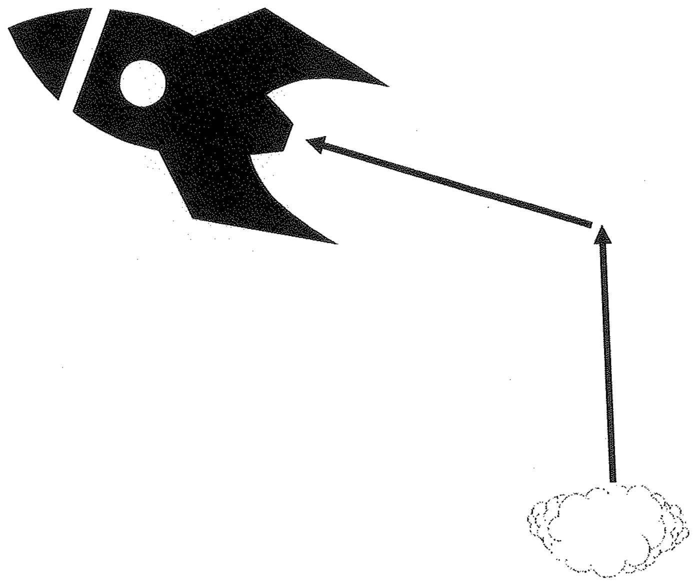
# Tough Luck!



**SIMILARITIES BETWEEN OVAA AND VERTICAL ANALYSIS**

**When launch angle is $90^0$, OVAA and vertical analysis are identical in every way.** Recall that OVAA launch rod formulas look like vertical formulas with $g * \sin\theta$ substituted for $g$. When the launch angle is $90^0$, $g * \sin\theta$ reduces to $g$, and the OVAA launch rod formulas are identical to vertical formulas. OVAA free flight formulas reduce to vertical formulas as well. Thus **vertical analysis is nothing more than a special case of OVAA. Significant non-ballistic character undermines OVAA, and it undermines vertical analysis no less. ALL single axis accelerometer analysis is perilous! Deal with it!**

**TESTABLE HYPOTHESIS**

The good news in all this is that **the constraint imposed by single axis accelerometers is ballistic trajectories, and not vertical trajectories.** <u>This is a testable hypothesis</u>. Here are some actual results from a flight of a *LOC VULCANITE*™ at $80^0$ – a large but not a terribly unusual angle for an accelerometer today. The stair stepped curve is barometric altitude against time. The line running substantially through it is OVAA altitude/time. The outlier curve is altitude/time from vertical analysis. (Note that vertical time of apogee is different from OVAA time of apogee.)



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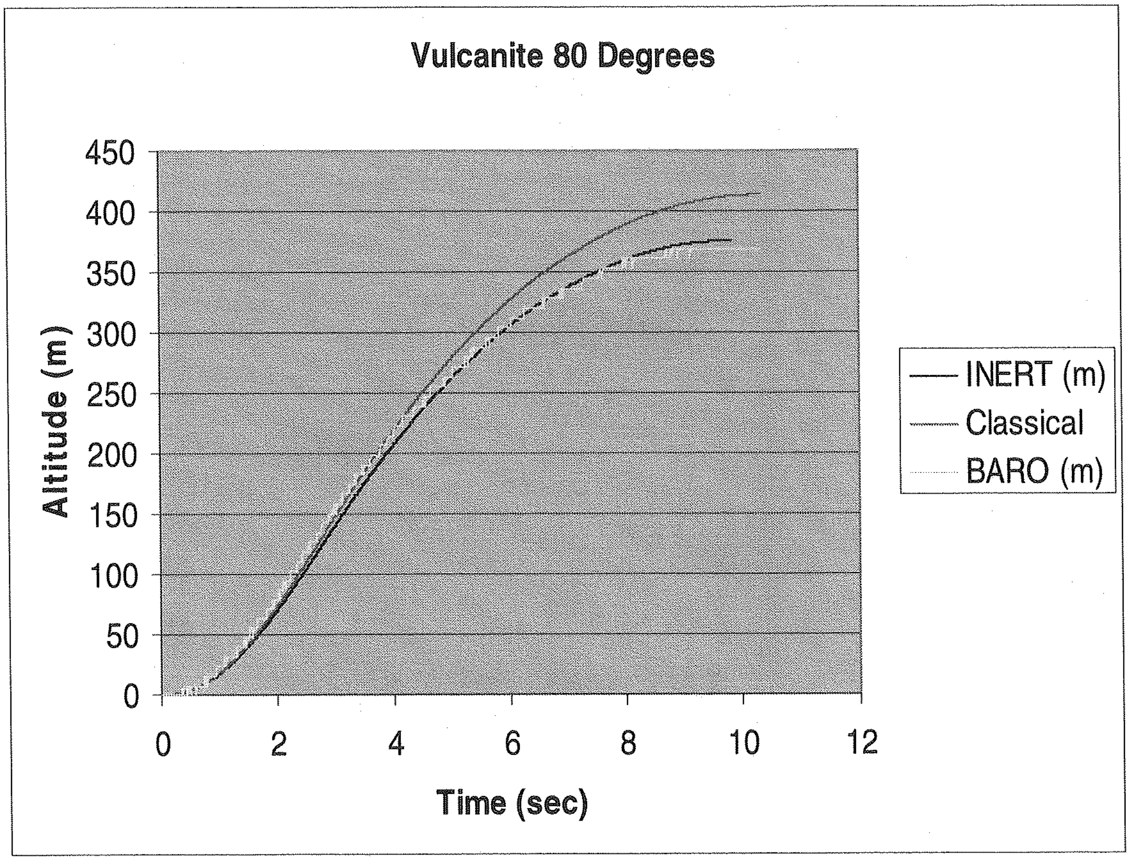


There is a bit of departure between OVAA and barometric altitude during high velocities in the boost phase. This is the signature of the Bernoulli effect, which we will revisit in the section on inertial/barometric closure.

## SECONDARY DATA FROM ACCELEROMETER ANALYSIS

Aside from primary trajectory statistics, accelerometer analysis can yield thrust/time curves and $C_d$/speed curves. These both come from the underlying principle

$$AccelerometerReading \approx ThrustAcceleration + DragAcceleration$$

Because this relationship holds at any angle with respect to the horizontal, off-vertical secondary data formulas are identical to the corresponding vertical formulas, except to the extent that they use velocity and altitude, which are themselves derived differently.

Drag coefficients are derived in the coast phase, where thrust is zero. In the coast phase, therefore, the entire accelerometer reading is drag acceleration. We must use velocity to find the coefficients and we must use altitude to reckon air density.

$$C_d = -\frac{2 * AccelerometerReading * BurnoutMass}{AirDensity * ReferenceArea * Velocity^2}$$



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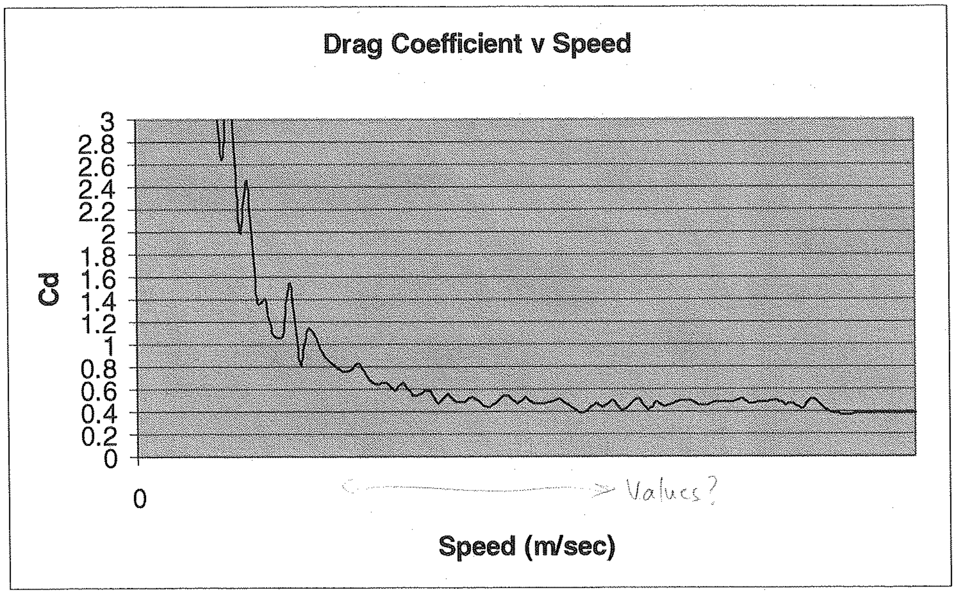
In the troposphere

$$AirDensity = BaseDensity * \left[1 - \frac{LapseRate * (Altitude - BaseAltitude)}{BaseTemperature}\right]^{\frac{HC}{LapseRate} - 1}$$

And

*BaseDensity* is air density at some standard altitude, *BaseAltitude*.
LapseRate is the Kelvin temperature lapse rate with altitude (6.5 Kelvins/km)
*HC is the hydrostatic constant*, $HC \equiv .03418155$ Kelvins per meter.

Here is a sample flight $C_d$ graph of a *LOC™ Vulcanite* from the experimental flights.




Note that velocity, here, should be air speed. Alas, accelerometers give us ground speed, and we have to make do with that.

Thrust is naturally derived in the boost phase, where a simple transposition yields

$$ThrustAcceleration \approx AccelerometerReading - DragAcceleration$$

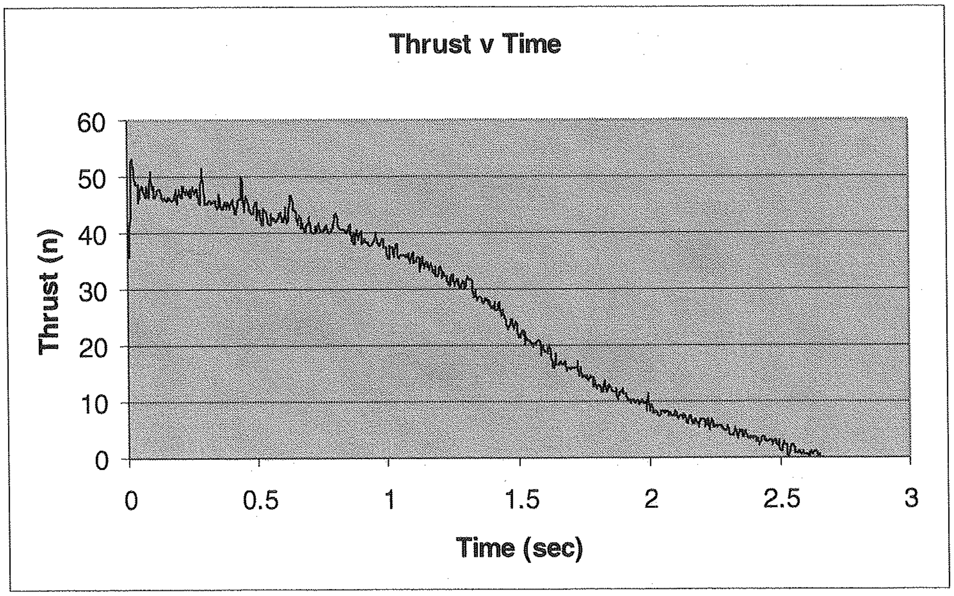
Since *DragAcceleration* is negative, we are essentially adding drag back into the accelerometer reading. To do this, we use velocity, altitude (for air density again), and drag coefficients found in the coast phase.

$$DragAcceleration = -\frac{C_d * AirDensity * ReferenceArea}{2 * Mass} * Velocity^2$$

and

$$Thrust = [Reading - DragAcceleration] * Mass$$

Here is a sample flight thrust curve from an Aerotech™ F25 motor used in the included experiments. The rocket was launched at $20^0$ off-vertical.




Drag coefficient curves usually have to be extended to higher velocities than those represented in the coast phase. Although cutoff velocity and maximum velocity are normally close, cutoff velocity may be somewhat lower for a number of reasons. The most common of these is that thrust tails off gently in many HPR motors. At some point, the rocket's speed dips below terminal velocity, and the rocket slows down a small amount even during boost. On might even envision an upper stage that slows down during its entire burn. There are other possible mechanisms, but we will not explore them here.

## OFF-VERTICAL TRAJECTORIES ASSUMED VERTICAL

Impulses from accelerometer flights have been historically short, and the shortfalls have frequently been attributed to trajectories that veer off from vertical and are nevertheless analyzed as if they were vertical. Let's examine that attribution.

The basic formula for thrust comes from the underlying principle, which holds at any angle with respect to the horizontal. The thrust formula is identical in the vertical and off-vertical cases, except insofar as it uses velocity and altitude, which are different in the two cases. Velocity and altitude are used in the drag term of the thrust equation. They are used explicitly, and implicitly in the evaluation of drag coefficient. Recall



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$$C_d = -\frac{2 * AccelerometerReading * BurnoutMass}{AirDensity * ReferenceArea * Velocity^2}$$

All this is computed in the coast phase, which is actually less vertical than the boost phase – and yet the trajectory is assumed vertical throughout. How does this affect drag coefficient? Let's look at the two terms in the denominator. The air density that we compute will be lower than the real air density, because we think the rocket is going straight up, and it isn't. The velocity that we compute will be lower than the real velocity, because we think the rocket is bucking the full brunt of gravity in a vertical ascent – and it isn't.

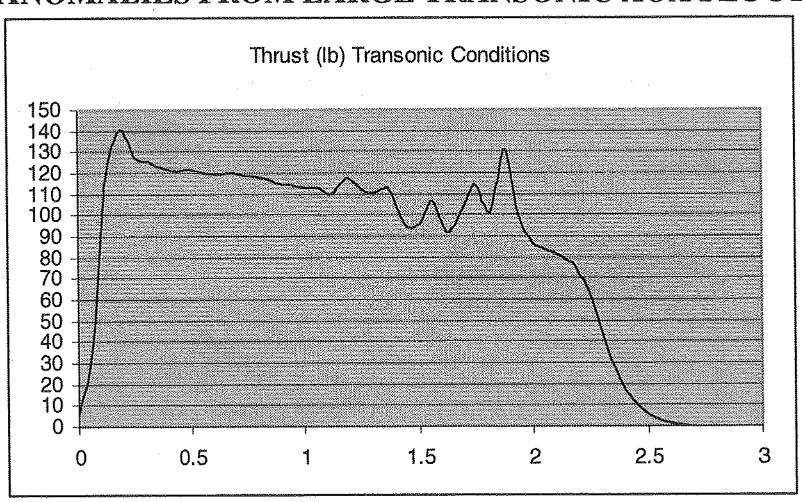
Both terms in the denominator are smaller than they should be, so **we overestimate drag coefficient. Consequently, we over-estimate drag acceleration in the boost phase. We add back more drag than we should in the thrust equation.** The result is that an off-vertical trajectory assumed vertical yields a slight OVERESTIMATION of thrust, and not a gross underestimation of thrust. The explanation for impulse shortfalls does not wash.

In fact, there are many possible explanations for these shortfalls including launch detect, launch rod friction, basic imprecision of flight data, and inaccuracy of particular programs. There is also the possibility that the shortfalls are, at least in part, real. This paper is agnostic on the subject, but it does present flight impulse values, which are only slightly short of standard values.

## TRANSONIC EFFECTS

The extension of drag coefficient to high velocities is normally harmless with respect to secondary data analysis, but it can introduce error when the rocket just reaches transonic velocities during the boost phase, and falls back to subsonic velocities by the time thrust ceases. In such a case, the derived drag coefficient curve is entirely subsonic, and extensions to transonic velocities are invalid. Thrust curve anomalies can result. Also, AOA can vary wildly in the transonic region.

## ANOMALIES FROM LARGE TRANSONIC AOA FLUCTUATIONS



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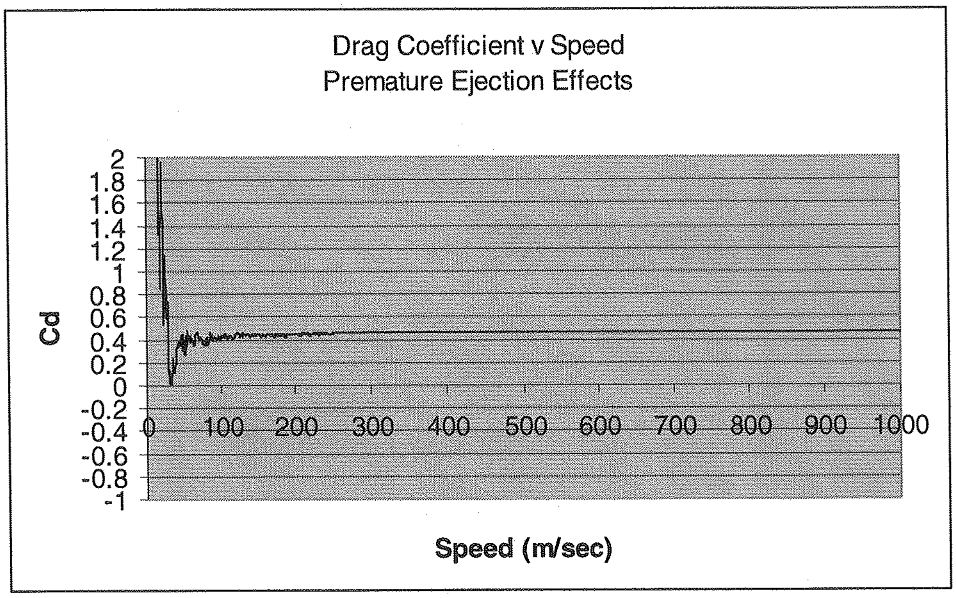
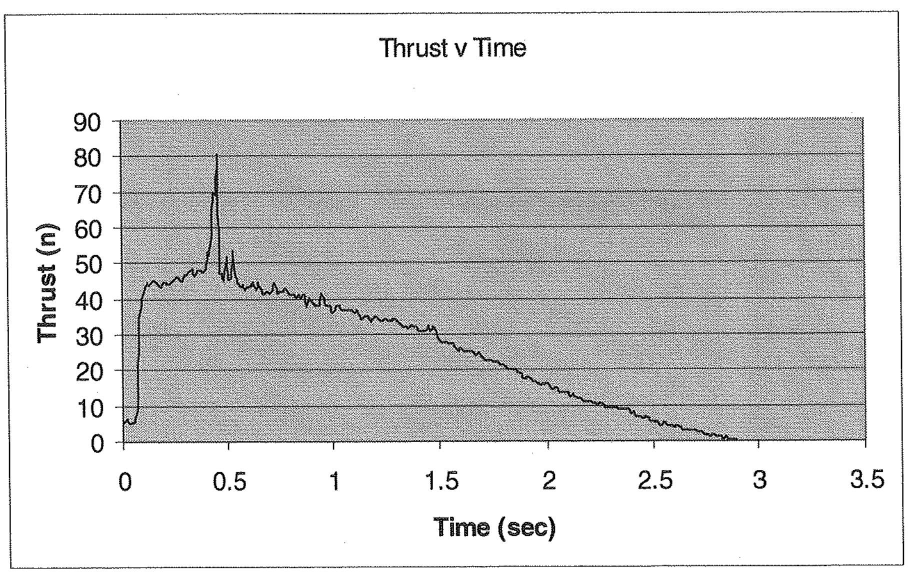
## PREMATURE EJECTION AND SECONDARY DATA

Flight computers are frequently used to activate recovery systems, and data collection must be rendered secondary to this mission. Sometimes, redundant instruments are used, and ejection is deployed by the first instrument to decide that apogee has been reached. That is, by definition, the decision at the earliest extreme, so fedundant instrumentation imposes a bias toward early ejection. There is also a human bias toward deploying precisely at apogee, and even without redundancy, one is likely to be slightly early half the time.

Consider that premature ejection usually propels the instrument compartment forward after burnout. The result can be a dip in the drag coefficient curve, and even negative $C_d$ values. The following data were collected by Brian Cole on the FC-877 flight computer.




There may also be complementary anomalies in the thrust curve, because drag is used to compute them. Here is such an anomaly in an F25 thrust curve.



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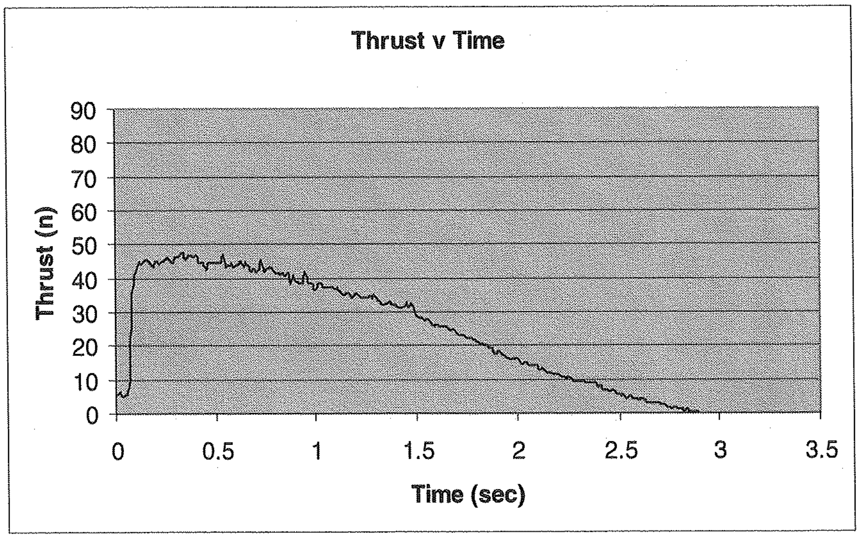
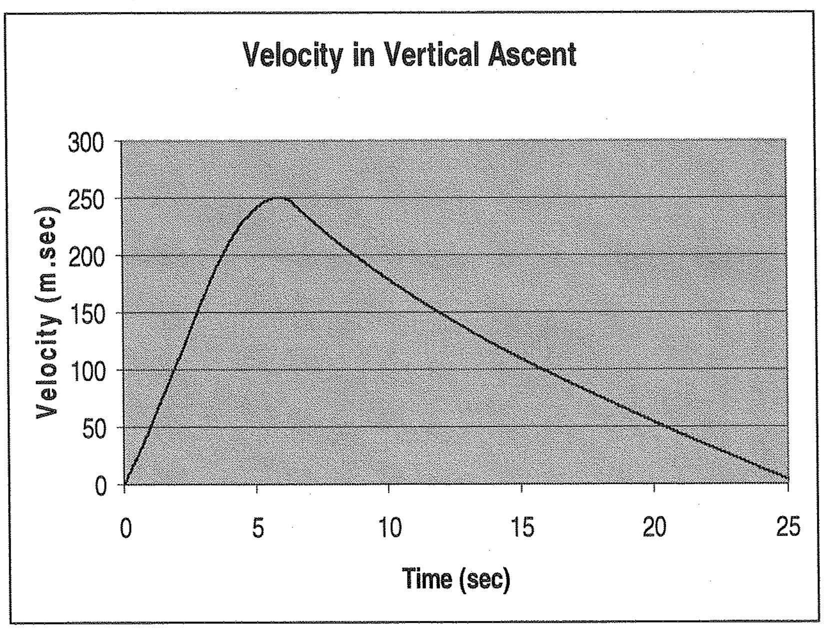
The best reference to detect this problem is the acceleration/time curve. Usually, the best remedy is to truncate the flight data just before ejection. It's not a perfect remedy, by any means. A better course is to set ejection for two seconds after apogee.

This particular thrust curve came from an ejection that was almost at apogee. The anomaly is at about the first third of the curve, because nose-over velocity was over 75 ft/sec, and not because the rocket was nowhere near apogee. Here is the same curve from data truncated just before ejection. In principle, the full trajectory can be extrapolated using this curve and the drag coefficient curve from the truncated flight.




## POSITIVE NOSE-OVER VELOCITY EFFECTS

Unlike vertical trajectories, off-vertical trajectories are characterized by nonzero velocities at nose-over. Here is a simulated velocity curve for a vertical flight.



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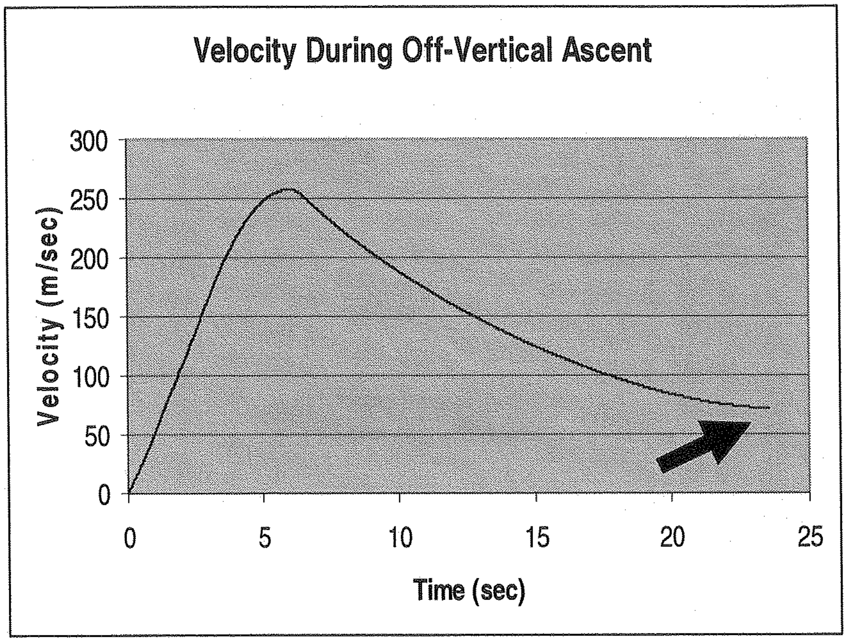
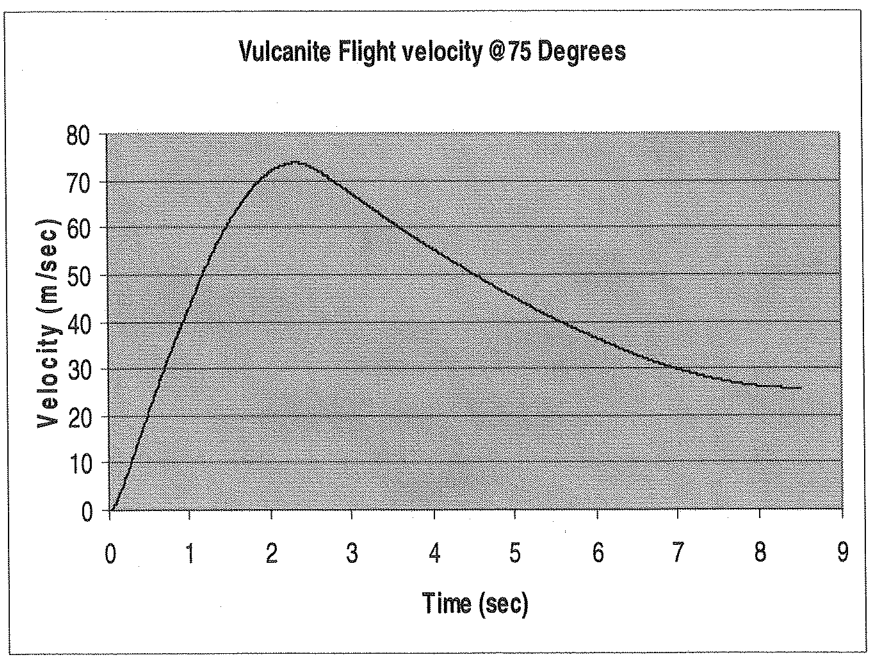
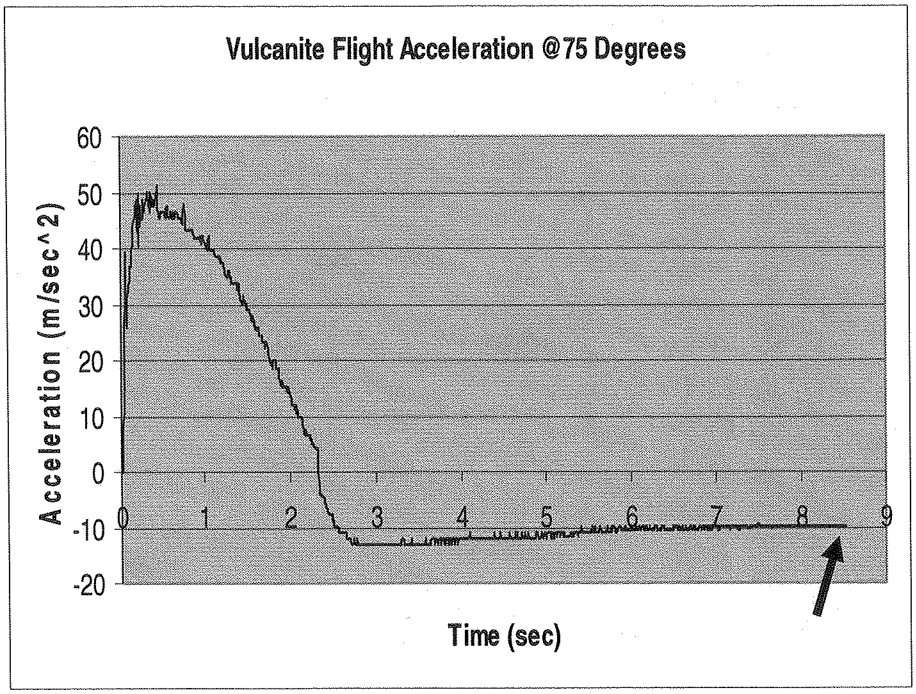
Here is the same rocket simulated in an off-vertical flight. Notice that the velocity at apogee is still positive.



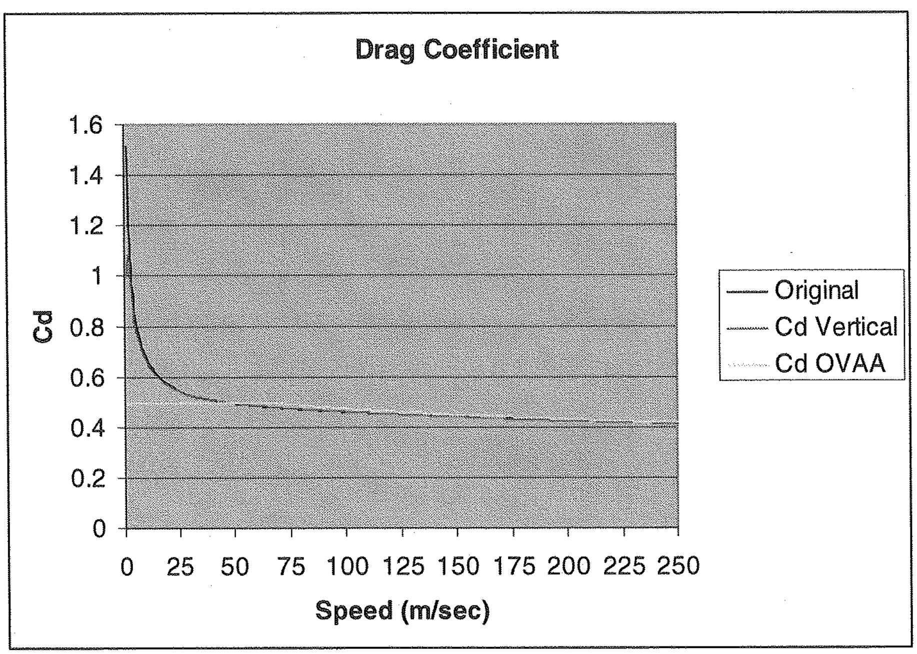

Here's a real flight velocity curve:




The acceleration/time curve from the same ascent shows some excess drag acceleration at apogee, corresponding to drag from nose-over velocity.




Since drag coefficients are derived in the coast phase, they can represent only those velocities encountered there. Nose-over velocity is the smallest velocity in the coast phase, and, again, this is nonzero in an off-vertical trajectory. The above graph depicts a computer experiment in which a $C_d$/speed curve is backed out of simulated trajectories, vertical and off-vertical. The curve from the off-vertical trajectory is flat, because it is missing information from speeds lower than the off-vertical nose-over velocity. Vertical trajectories are recommended for full $C_d$ curves.



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# Part II: Inertial/Barometric Closure (IBC)

**BAROMETRIC ALTIMETERS**

Barometric altimeters respond to altitude-related pressure differentials, which are caused by nothing but gravity. Accelerometers are therefore oblivious to the very force that drives altimeters. Data streams from these two technologies therefore provide independent, observations of the same flight.

Barometric altimeters are predicated on three principles:

1) Hydrostatic equilibrium;
2) The ideal gas law; and
3) The first law of thermodynamics.

Of these, hydrostatic equilibrium holds least well. It is predicated on the idea of still air that remains still because the weight of a column of air at any level is balanced by the pressure of the column from below. Tropospheric air is not still.

The ideal gas law holds reasonably well, except where phase changes occur, as they do in cloud banks. It is not uncommon to see blips in balloon data as clouds are traversed. Hydrostatic equilibrium is a weak link. Water content also complicates temperature lapse rates that would otherwise emerge from the three principles.

The principles lead to a formula already presented, which is the underlying principle of altimeters, in thr troposphere (or more generally with nonzero temperature lapse rates):

$$Altitude = \left(\frac{BaseTemperature}{LapseRate}\right) * \left\{1 - \left[\frac{Pressure}{BasePressure}\right]^{\frac{LapseRate}{HC}}\right\} + BaseAltitude$$

And

*BaseTemperature* is the Kelvin temperature at some standard altitude, *BaseAltitude*.
LapseRate is the Kelvin temperature lapse rate with altitude (6.5 Kelvins/km)
$HC$ *is the hydrostatic constant*, $HC \equiv .03418155$ Kelvins per meter.

This represents the troposphere, where the temperature lapse rate is observably constant under normal circumstances. In the stratosphere, the lapse rate goes to zero an ultimately reverses. One way of defining atmospheric layers, in fact, is by lapse rate patterns. For zero lapse rate,

For zero lapse rate, the relationship is:

$$` \; Altitude = \left(\frac{Temperature}{HC}\right) * \ln\left(\frac{BasePressure}{Pressure}\right) + BaseAltitude$$



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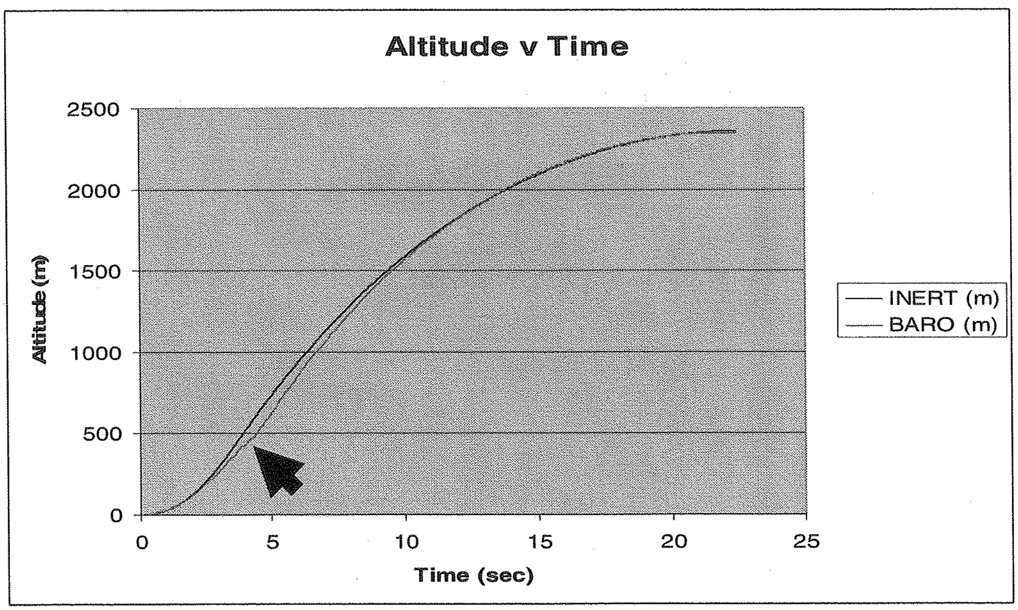
We will be concerned primarily with the first equation, since most HPR flights occur in the troposphere.

## ALTIMETER QUIRKS

Here are some signatures of known altimeter quirks. They come from aerodynamic effects at the static port, and they involve size or placement of the static port. All of these effects are to be found in regions of high velocity.

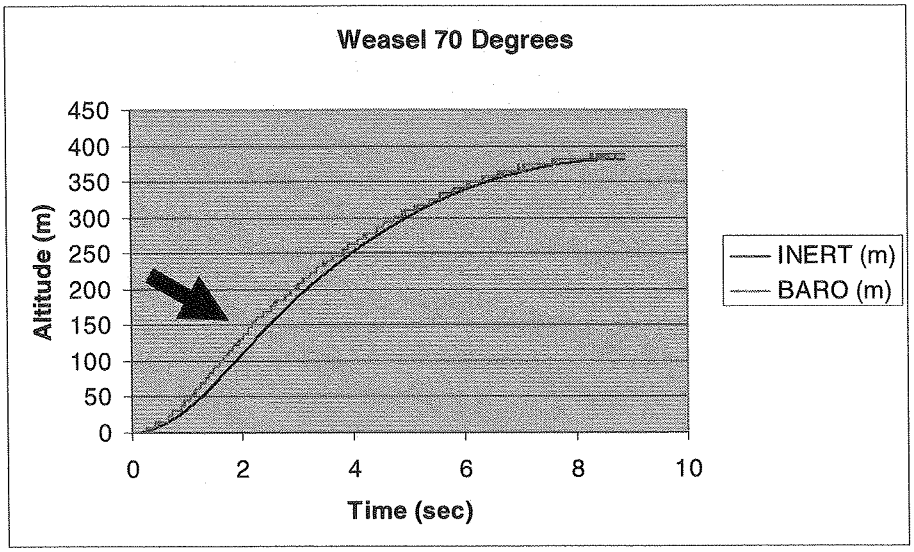
Here is altimeter delay, which comes from static ports that are too small. The effect presents as a pot belly in the barometric altitude curve near maximum velocity.

Altimeter Delay




Here is the very opposite: the Bernoulli effect. This results from placement of the static port in a region where local pressure is lower than ambient pressure, because of high airspeed.

# Bernoulli Effect

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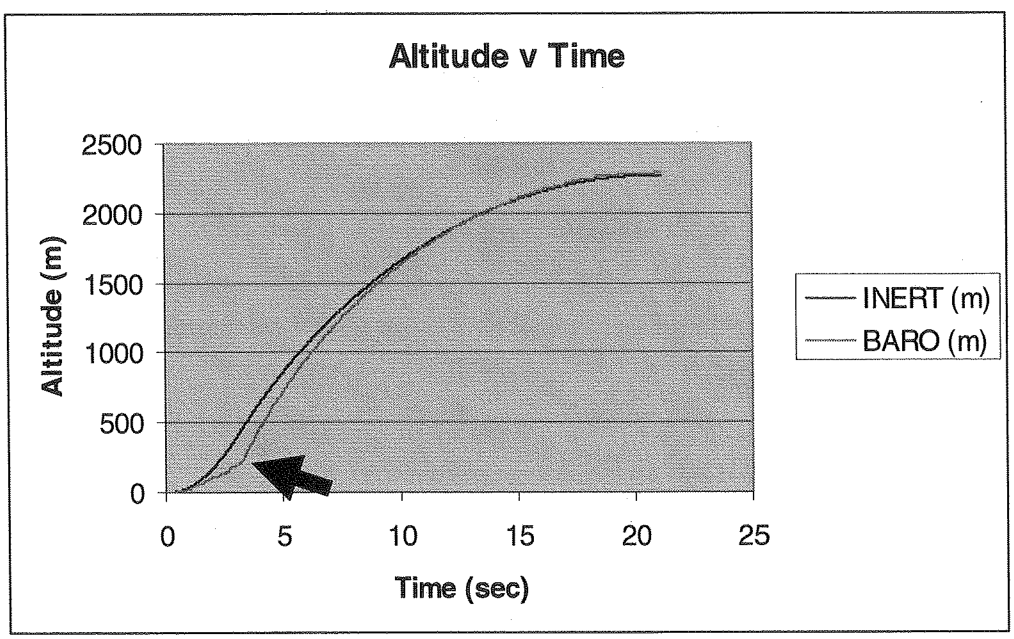




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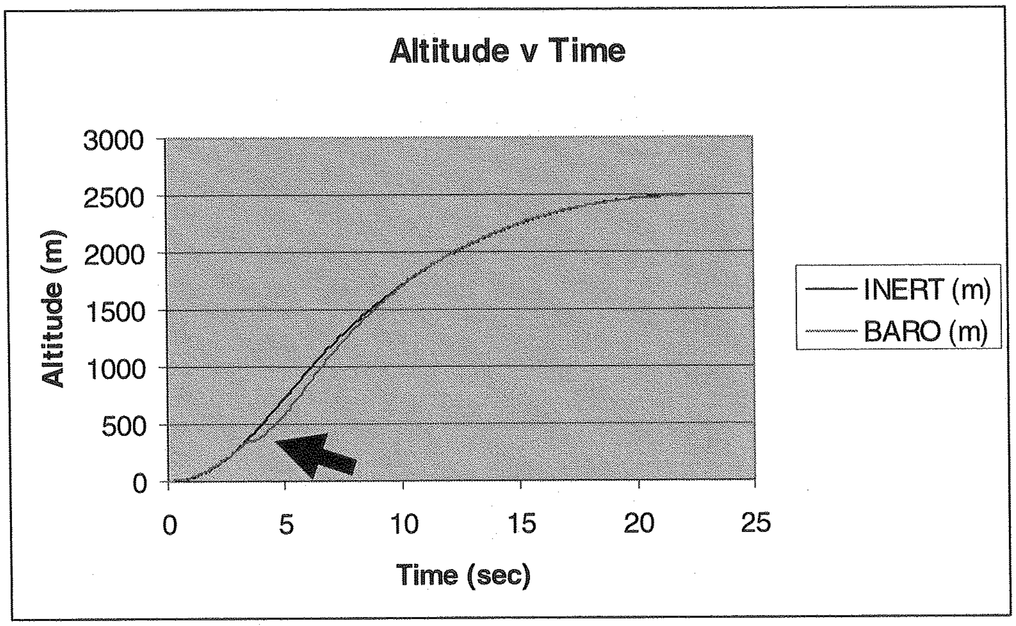


## COMPARING BAROMETRIC AND INERTIAL DATA

Barometric and inertial altitudes can be compared in a number of ways. Some are

1) Altitude at apogee (Closure);
2) Graphical comparison of altitude/time curves (as above!); and
3) Statistical comparison (using $R^2$ and mean squared error)

Of these, item two is self-explanatory, and is illustrated in the *Altimeter Quirks* section above.

## INERTIAL/BAROMETRIC CLOSURE (IBC)

This involves the simple comparison of inertial and barometric altitudes at apogee. In the entire breadth of the altitude/time curves, this method uses only two points, so it is easy to under-value it. Actually, **IBC is vital!** The flight computer delivers two altitudes. If they don't resemble each other, then something is wrong. If they rarely resemble each other, something is busted.

In fact, IBC hasn't worked very well in the past, and rocket enthusiasts don't talk about it much. Some hardware providers don't even display barometric altitude in their accompanying software, for fear of embarrassment.



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## REASONS FOR IBC FAILURE

Here are some testable hypotheses concerning reasons for closure failure.

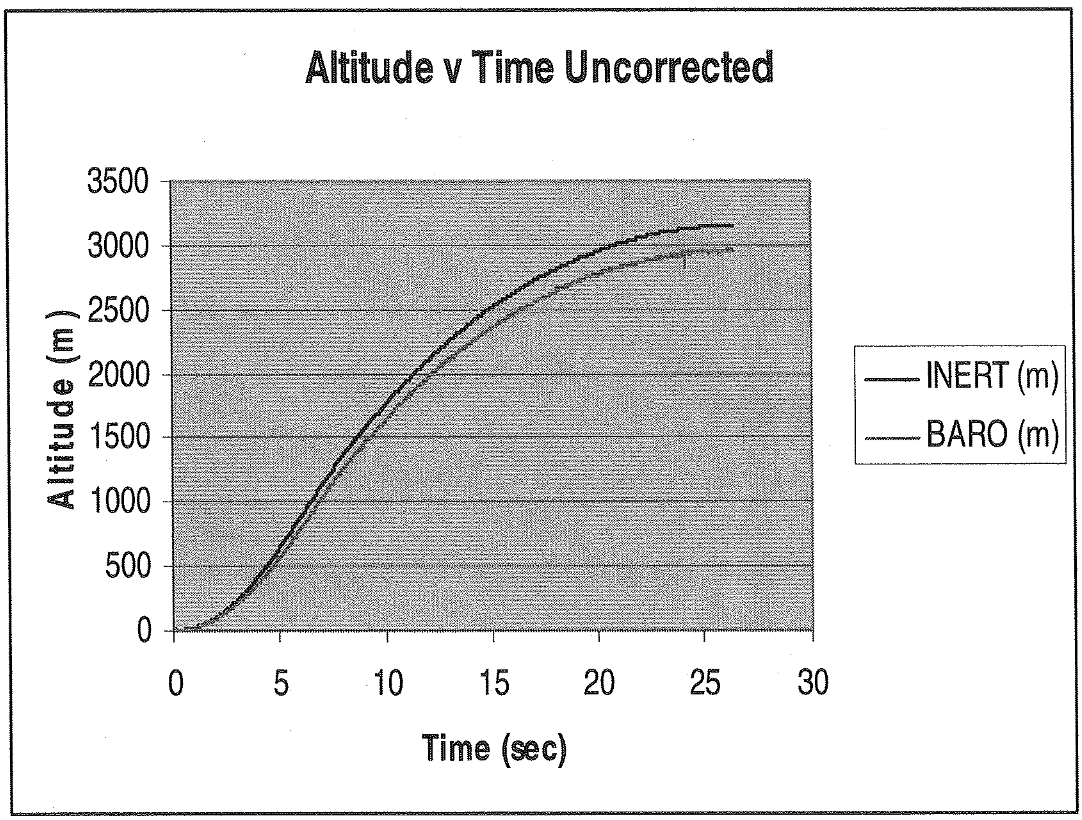
1) Off-vertical trajectories;
2) Lack of altimeter temperature correction;
3) Self-calibrating accelerometers.

We have already addressed item 1, since OVAA is a proposed remedy.

## ALTIMETER TEMPERATURE CORRECTION

If you look back at the altimeter altitude equations, you will notice that both begin with a multiplier of Kelvin temperature. In the case of the tropospheric equation, this is the temperature at some reference altitude. Most rocket altimeters use launch level as the reference altitude. Airplane altimeters use sea level. Clearly, the value must be important. It is possible for an altimeter to sense temperature and correct accordingly. Most rocket altimeters do not.

The following altitude/time graph is taken from an ARLISS flight at Black Rock by Geoff Huber. Archived temperature data for the region suggest the temperature was about 94$^0$F. Here is the graph of inertial and raw altitude.



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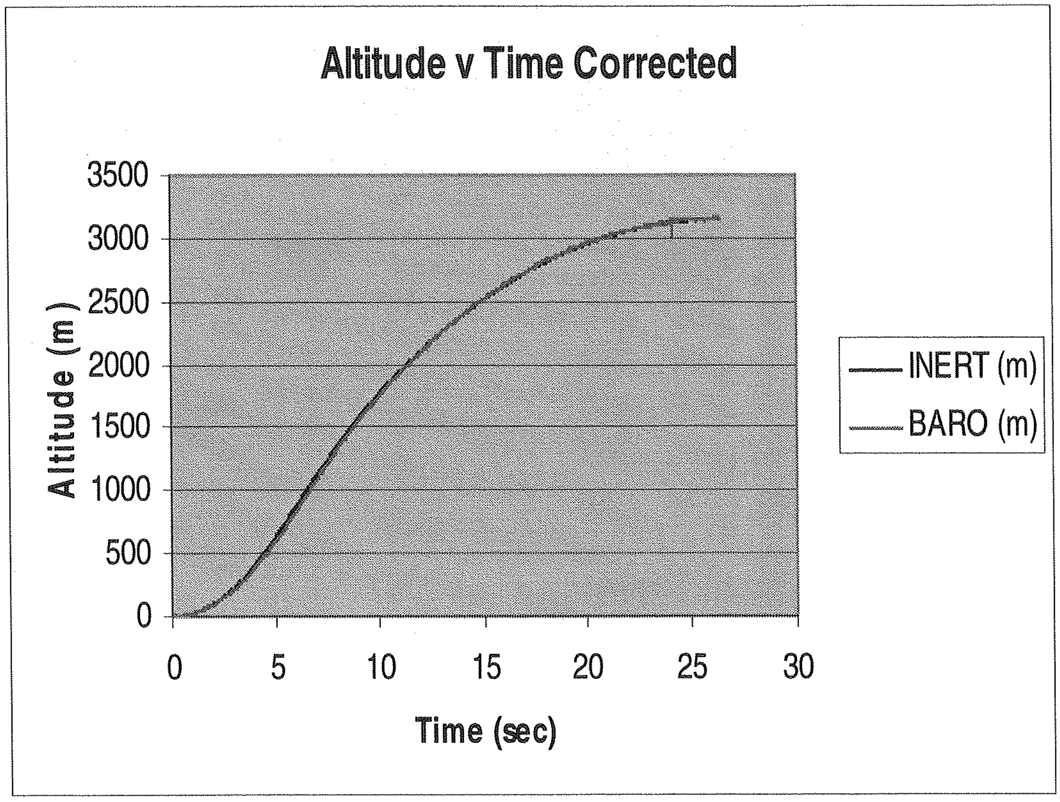
Here is a graph of the same flight with barometric data corrected for temperature.




For instruments using the launch site as the reference, this source of error is easily eliminated, in tropospheric flights, using the formula:

$$CorrectedAltitude = AltitudeReading * \frac{(273.15 + CelsiusRealTemperature)}{288.15}$$

If the altimeter is calibrated at some other altitude, say sea level, then temperature correction consists in two steps (Reference 1)

1) Find the expected absolute temperature at BaseLeve, level; and
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Step 1

$$ExpectedBaseTemperature = (GroundAltitude - BaseAltitude) * LapseRate + GroundTemperature$$

Where LapseRate= .0065 Kelvins per meter

Noter that if $GroundAltitude = BaseAltitude$, then
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Step 2

$$CorrectedBarometricAltitude = \frac{ExpectedBaseTemperature}{288.15} * UncorrectedBarometricAltitude$$

Where temperatures are in Kelvins.

Thus, on days warmer than 15 degrees Celsius (59 degrees Fahrenheit), altimeters underestimate altitude; on days cooler than that, altimeters tend to overestimate altitude. When flight computer data are taken at face value, lack of temperature correction can cause IBC failure on hot or cold days. The experimental section of this paper examines the effect.

## SELF-CALIBRATING ACCELEROMETERS

At one time, a rocket enthusiast would have to calibrate her accelerometer immediately before launch by holding it upright. The procedure was inconvenient, and the calibration was subject to drift between calibration and launch. More recently, accelerometers were made self-calibrating. When they are turned on, they assume they are vertical. They take base line readings in a circular buffer, and these readings serve as a working value for $g$.

When flight data are taken at face value, self-calibrating accelerometers magnifies errors from off-vertical trajectories, because what the instrument thinks is $g$ is actually $g * \sin\theta$, where $\theta$ is the launch angle. It is possible to recalibrate the data when the launch angle is known, but accurate launch angles are very difficult to set – particularly with launch equipment provided at public events. Self-calibrating accelerometers also magnify this error, and it is impossible to fully capture the effect after the launch. Therefore it is reasonable to speculate that closure is a more serious problem since the advent of self-calibrating accelerometers than it once was. This paper examines the size of such effects.

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No matte what we do, closure will be imperfect. Two altimeters will fail to comare perfectly under the exact same conditions.

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We now consider statistical comparison of the two altitude/time curves. Mean square error (MSE) and a nonlinear extension to $R^2$ are to be recommended for the purpose. The latter statistic is frequently used in nonlinear regression analysis. Its formula is

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This is evaluated from launch to maximum inertial altitude.

The numerator is the sum of the squared error terms from barometric altitude in the inertial data. The denominator is the sum of squared deviations from the mean barometric altitude in the barometric data. If the inertial altitude values were derived from a least squares fit in a regression on barometric altitudes, then this would be the standard $R^2$



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statistic, and the possible range would be [0-1]. Standard variance partitioning does not hold in the nonlinear, untransformed context, however, and the true range of this statistic is unbounded on the left. The maximum is still unity, and in practice, negative values indicate an objective mistake. In a reasonable OVAA correspondence, the numerator is dwarfed by the denominator.

**ANGULAR BACKTRACKING**

Statistical comparison is most useful in computer applications. For example, we can ask a computer program to vary the launch angle to minimize MSE or to maximize $R^2$, thereby optimizing the correspondence between barometric and inertial altitude/time curves. The angle, so backtracked, is potentially useful in three contexts:

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Item 2 is also controversial, because (like item 3) the validity of the results is not provable. Item 1, which is provable and constitutes another form of closure, is performed in this paper. Items 2 and 3 are illustrated.

Subtleties arise in the context of self-calibrating accelerometers. If we are trying to test our model, we must recalibrate the data at each new angle tried, because the calibration angle and the angle we are seeking are the same. This is also true when we are trying to refine the actual launch angle. When we are trying to repair tip-off, however, the equivalent angle we are seeking is very different from the calibration angle. We should use the intended launch angle for the calibration angle there.

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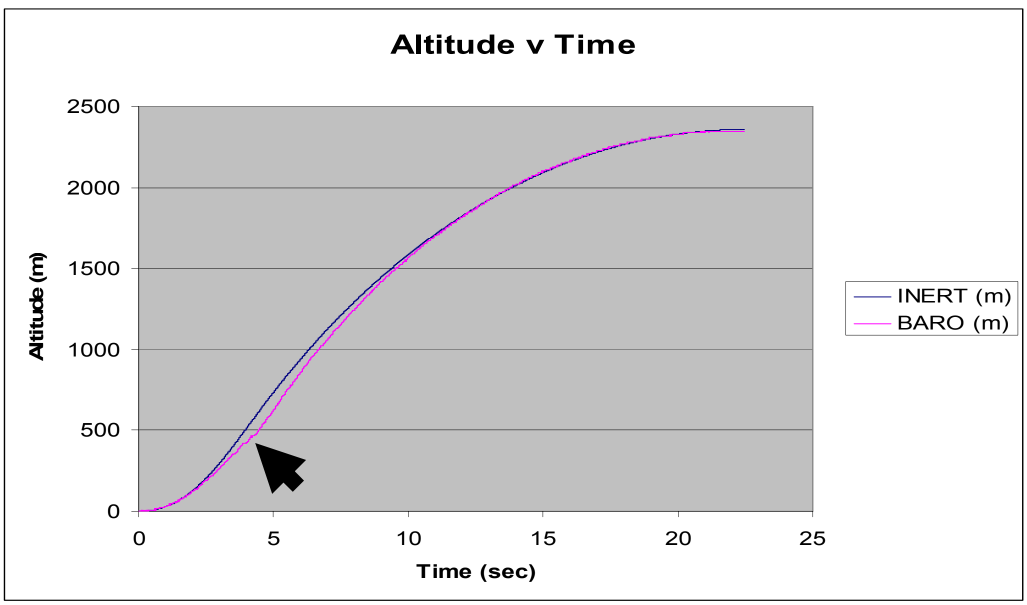


Below is the very opposite: the Bernoulli Effect. This results from placement of the static port in a region where local pressure is lower than ambient pressure. In the graph below, the decrease in error with altitude appears slower. That's because the launch was at $70^0$. The nose-over velocity is substantially positive, and velocity is higher all along this curve than it is along the other curves in this section, which are from flights that are more vertical.

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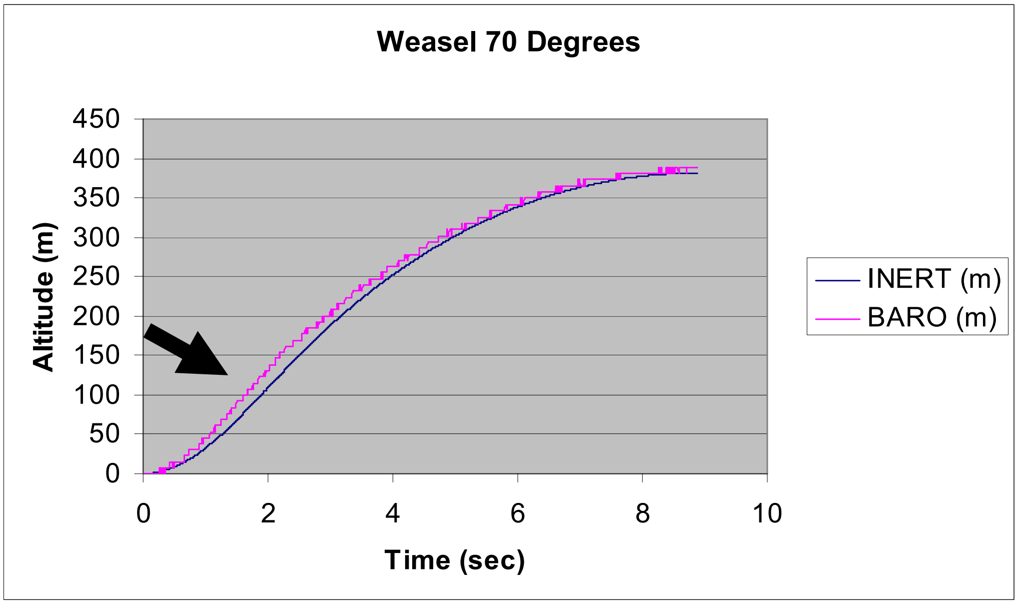


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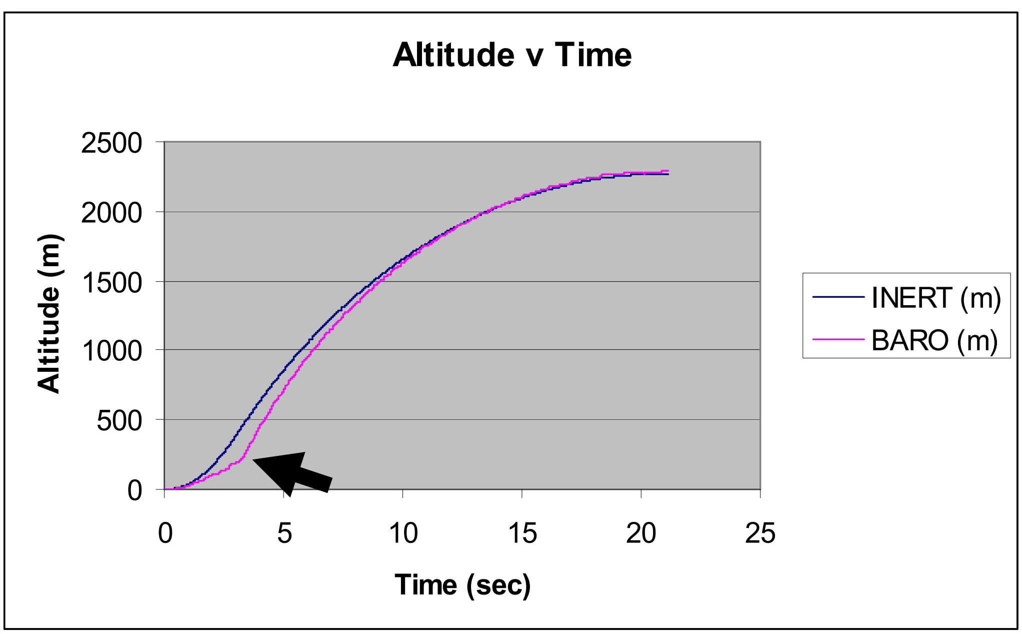

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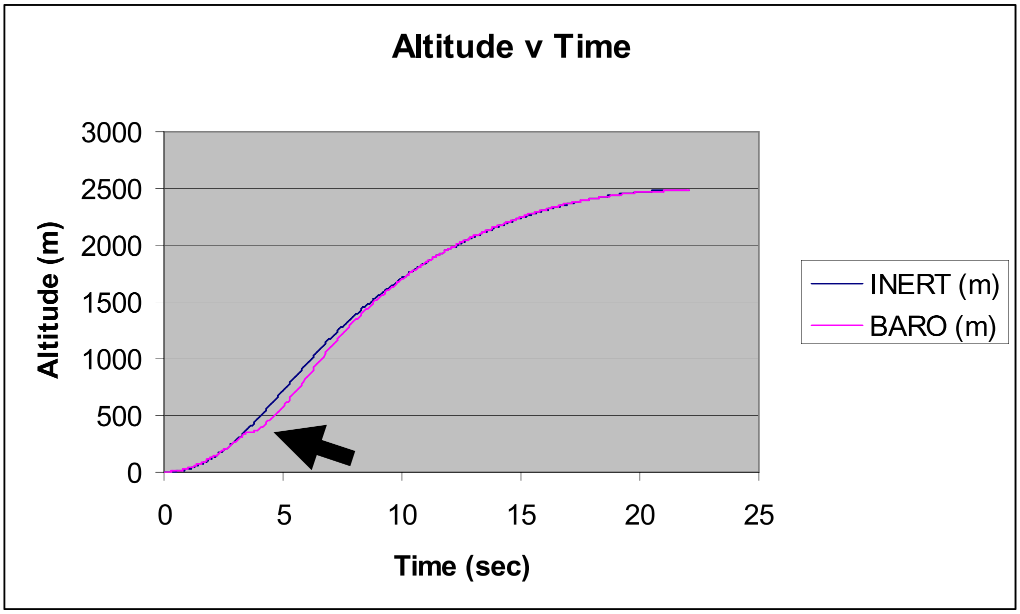


For a more detailed discussion of altimeter errors, see Appendix C.

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Here are some testable hypotheses concerning reasons for closure failure.

1) Deficiencies in accelerometer analysis
2) Off-vertical trajectories;
3) Lack of altimeter temperature correction;
4) Self-calibrating accelerometers.

We have already addressed item 1, since OVAA is a proposed remedy.

**DEFICIENCIES IN ACCELEROMETER ANALYSIS**
These are evident from discussions in the last section. They boil down to one essential point: <u>Accelerometers do not detect apogee per se; they estimate statistics at every point by dead reckoning, and apogee is just another point</u>. We identify the point of apogee after the fact as the point that happens to have the highest associated altitude. If the underlying analytic assumptions are incorrect (for example, it the trajectory is not substantially ballistic), then the point's statistics are incorrect, and the point itself may be incorrectly identified.

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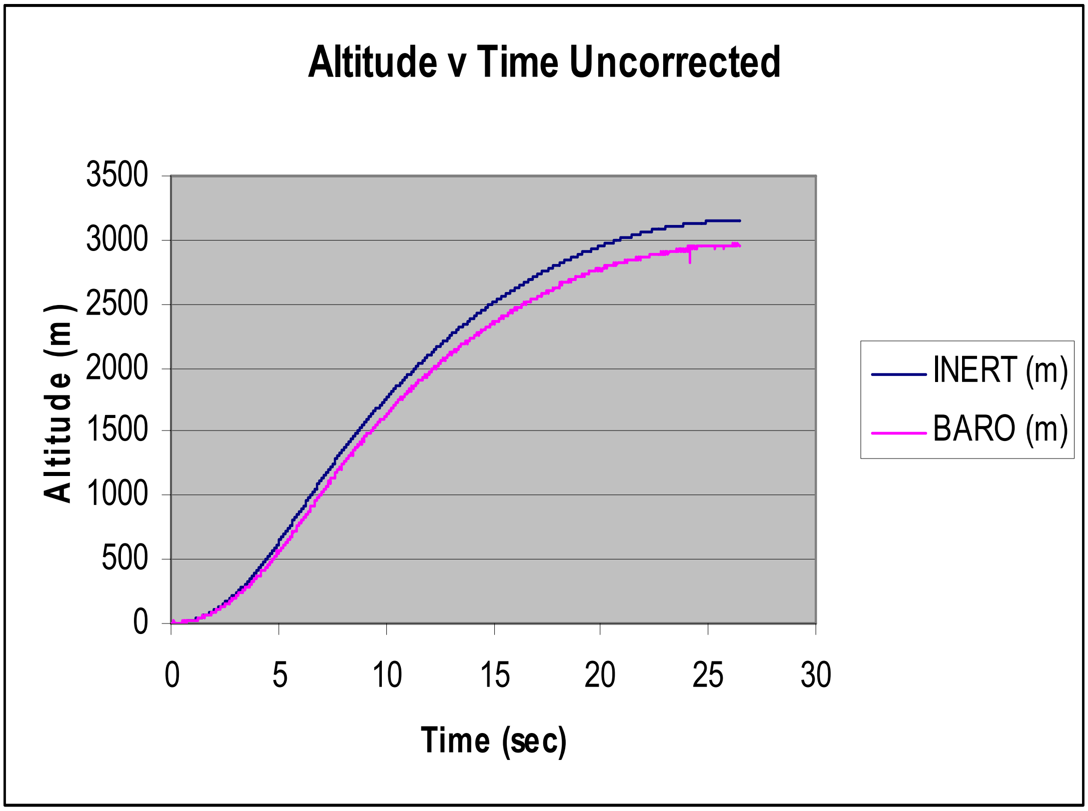


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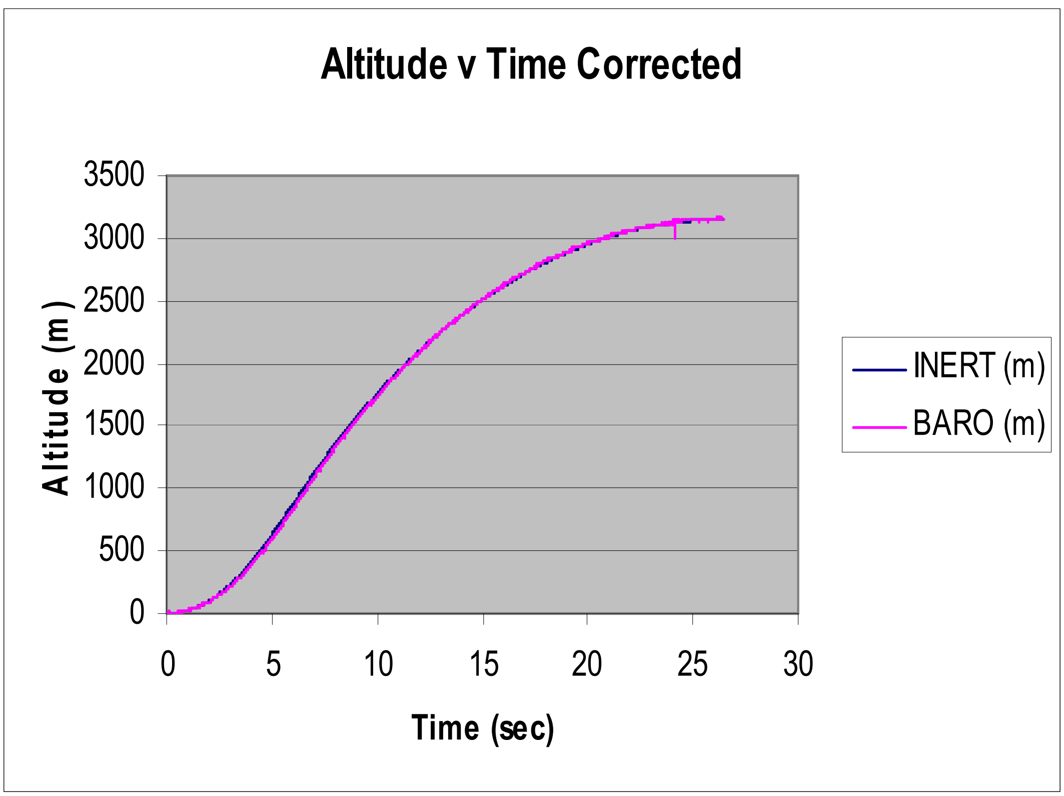

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This is evaluated from launch to maximum inertial altitude.

The numerator is the sum of the squared error terms from barometric altitude in the inertial data. The denominator is the sum of squared deviations from the mean barometric altitude in the barometric data. If the inertial altitude values were derived from a least squares fit in a regression on barometric altitudes, then this would be the standard (linear) $R^2$ statistic, and the possible range would be [0-1]. Standard variance partitioning does not hold in the nonlinear, untransformed context, however, and the true range of this statistic is unbounded on the left. The maximum is still unity, and in practice, negative values indicate an objective mistake. In a reasonable OVAA correspondence, the numerator is dwarfed by the denominator.

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Statistical comparison is most useful in computer applications. For example, we can ask a computer program to vary the launch angle to minimize MSE or to maximize $R^2$, thereby optimizing the correspondence between barometric and inertial altitude/time curves. The angle, so backtracked, is potentially useful in three contexts:

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This paper will examine the accuracy of backtracked launch angles. The computer program used, *OVAA2.xls*, performs angular backtracking automatically by means of the *Excel* solver.

**BACKTRACKING SESNITIVITY**

One point worth mentioning is that angular backtracking is least accurate in launches intended to be vertical. Some of the difference between inertial and barometric altitude is caused by factors that have nothing to do with launch angle, but angular backtracking attributes all differences to launch angle. Suppose a minor altitude difference arises in a vertical launch. Small launch angle deltas from vertical make very little no difference in altitude, so a substantial angular deviation will be required to account for the observed difference that has nothing to do with launch angle. Thus, altitude differences that are really related to launch angle will be properly accounted, but the component of divergence that is not altitude related can make the backtracked angle inaccurate. By contrast, a small angular difference near $45^0$ makes a large difference in altitude, so altitude divergences that are not truly related to launch angle do not affect backtracked angle much in that region.

Thus backtracked launch angles are more apt to be importantly affected by random divergences if the intended launch angle is near vertical. This fact is evident in the experimental results section.

# Part III: Experimental



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## RESEARCH QUESTIONS

1) **How well does OVAA work in practice?**

2) **How do backtracked launch angles compare with intended launch angles?**

3) **How does OVAA analysis compare with vertical analysis of off-vertical flights?**

4) **Does temperature correction of altimeter data contribute to inertial/barometric closure?**

5) **In vertical analysis, what is the effect, on data from off-vertical trajectories, of not compensating for self-calibrating accelerometers?**

6) **In OVAA analysis, what is the effect, on data from flights with launch angle errors, of accelerometer self-calibration?**

7) **How accurately can motor impulses be determined from flight data?**

8) **Can launcher tip-off be remediated through angular backtracking?**

## THE ROCKETS

Three rockets were used in these experiments:

1) A stock *LOC Vulcanite*™
2) A stock *LOC Weasel*™; and
3) A modified *Aerotech Initiator*™

The third rocket, which I shall refer to as a *Stretch Initiator*, is a standard *Initiator* fitted with an instrument bay of the same diameter and a slightly shorter nose cone. Some of the decisions made in this project are explained best by history, and each rocket has its own. Therefore, a synopsis of each rocket is included below.

## WEASEL

The Weasel's maiden flight was on 04/21/2007 at the Amherst, Ohio field of the *Skybusters* rocket club. Since the field was large, and I was eager to get very off-vertical data, I launched it at $70^0$ on an Aerotech F25-6. Winds were about 10 MPH, and the launch was into the wind. The motor deployed recovery 2 seconds sooner than the advertised delay time. In addition, the static port for the altimeter proved somewhat small, and altimeter delay was evident in the RDAS flight computer data (200 sps). Those data were reassuring and serviceable, but imperfect for the above reasons. The Amherst field is about 2.5 hours drive from my house, and I arrived too late for a second launch that day.



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I returned on 05/06/2007 for a two-day launch in the same location. I arrived late on the first day, and the launch was almost immediately shut down because of 25 mph winds. The next day, winds were 10-15 MPH. I had intended to launch into the wind at $80^0$, but all of my wits had not recovered from a night in a motel, and I accidentally launched at $70^0$, because I somewhere had it in my head that vertical was $100^0$, so that $80^0$ should be $20^0$ off-vertical. This I realized just as the rocket took off on an F25-9. I had used the long delay because of the previous short delay, but this time, the delay was longer than advertised, and the rocket disappeared somewhere in a field beyond an orchard. 3.5 hours of searching yielded no trace of it.

That was the season's last day for the Amherst field, since the owner was about to plant. So, evidently, were other farmers, because the rocket turned up within days. It was returned to members of the *Skybusters* club when they presented the owner with a certificate of appreciation about two months later. In an act of great kindness, they mailed the vehicle back to me. The RDAS flight computer and flight data (200 sps) were still intact.

On 07/15/2007, I launched this rocket two more times at the *Lutherlyn* field of the *Pittsburgh Space Command*. Both launches were on F25-9's. One launch was vertical and one at $80^0$ – to my limited accuracy. Winds were about 6-8 MPH, and I asked the RCO to launch in moments of relative calm. Launches yielded good data, but three features of those launches are noteworthy.

First, the Lutherlyn field is not as large or symmetric as the Amherst field. Launches always take place from the same station and in pretty much the same direction. One cannot arrange to launch into the wind. On this occasion, the wind was going mostly with the rocket, and of course it was not perfectly with the rocket either.

Secondly, I had one F25-9 left over from the last Amherst launch, and had been searching for a second. I found one at a local hobby shop, and was so overjoyed to find it that I did not notice that the two motors had different vintages. The F25, G40, and G80 motors had recently been redesigned (see discussion below), and the impulses of all three are now lower than they had been. The F25-9 I found at the local hobby shop was of the earlier vintage, and yielded a different impulse value, which was nevertheless consistent with those of similar motors. Note that motors of different vintages have different outward appearances. Pictures of the two specimens are included in the section on motors below.

Finally, the delay on the F25-9 was about a second short, and ejection was premature, but only very slightly. The resulting thrust curve is illustrated in the section in premature ejection. The curve was remediated by simply truncating the data at the point of ejection, which was almost exactly at apogee anyway. The remediated curve is also presented there.

**VULCANITE**

While the Weasel was AWOL, I needed substitute hardware, so I pulled a Vulcanite from the shelf. I had flown this model for some years. It has a long ¼ inch launch lug, which is



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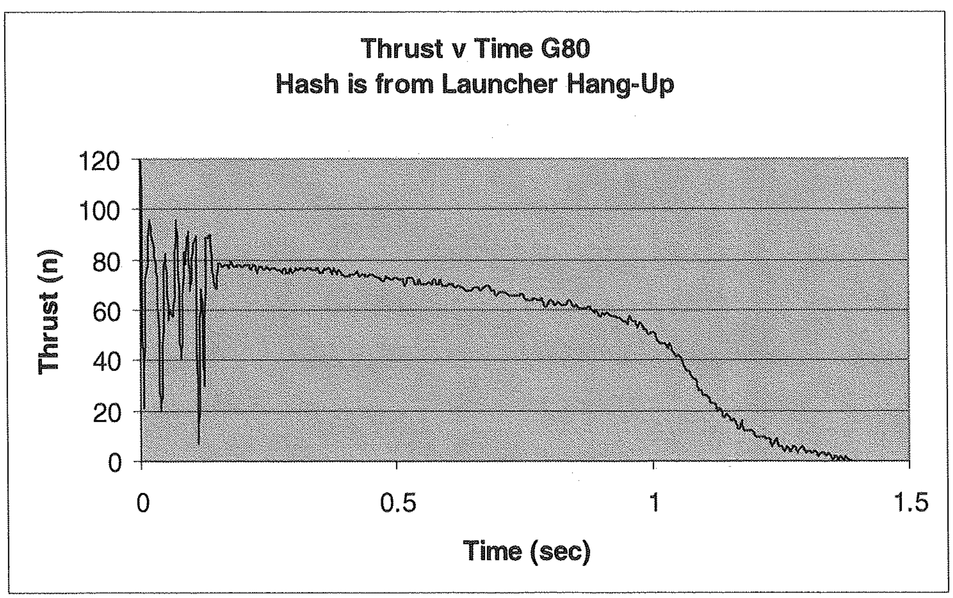
unforgiving of corroded rods. I used it several times to test the RDAS flight computer in the fall of 2006, but had trouble with the rocket snagging during launch. With the Weasel AWOL, I had to fly this rocket again at the Lutherlyn field of the Pittsburgh Space Command, and I judged that a $70^0$ angle might be stretching things. Therefore I chose $75^0$, $80^0$ and $90^0$ angles. These flights took place on the afternoon of 06/16/2007, and used an *OZARK ARTS* flight computer sampling at 100 sps. Motors were Aerotech G40's of latest vintage.

Both off-vertical launches went well. The vertical launch tipped off badly and landed in a lake on the other side of a wood at the edge of the field. By insane good luck, the rocket landed just off shore, and its long shock cord snagged on a tree limb. This circumstance prevented it from sinking or drifting. Miraculously, the flight computer was functioning, and it delivered up data – but not of a vertical flight. The data from this flight are used, here, to demonstrate remediation of data from flights marred by early problems like whip and tip-off.

The vertical flight was the last of the three anticipated launches, and the rocket was good for no more flights on that day. The waterlogged motor adapter still refuses to give up (what are now the shards of) the spent motor.

Reluctantly, I substituted a 2006 dataset for the vertical flight, even though it snagged badly on the launch rod, and even lifted the launcher briefly from the ground. The motor for this launch and all the 2006 launches was the G80. The first half of the thrust curve is hash, but the drag coefficient curve, taken from the coast phase, is intact. The impact of the event is a very short altitude (barometric and inertial) and a short impulse, but little else.




Winds on the afternoon of 06/16/2007 were 0-3 MPH – Rocket Weather!

## STRETCH INITIATOR

This was also a rocket off the home shelf, and it was also launched on06/16/2007 at Lutherlyn, but it was launched in the morning. It had been purchased just before Aerotech's fire, and it arrived with an asymmetric nose cone. Rather than bother the company at that particular moment in their history, I substituted a similar, though shorter, nose cone. I also added an instrument bay.

Winds in the mid-morning of 06/16/2007 were variable at around 5-10 MPH. The rocket was launched with winds coming almost directly from the side. Data were serviceable, but not as good as data for the other rockets. This fact may have had to do with the wind, or the lower stability margin from the elongation.



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**ROCKETS USED IN THESE EXPERIMENTS**
**L TO R: WEASEL, STRETCH INITIATOR, AND VULCANITE**



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Motors for this rocket were Aerotech G40's of latest vintage. The flight computer was also an *OZARK ARTS* sampling at 100 sps.

## MOTORS

Motors were intended to be reasonably homogeneous. The Weasel used Aerotech F25's; the other rockets used Aerotech G40's. Unbeknownst to me, however, the motors had been recently redesigned, and the impulses had decreased. The new impulses are reflected in the Aerotech catalog, but not all are reflected in NAR S&T reports. I have seen only the retest of the G40.

Newer vintage motors are easily recognizable by a yellow slug at the forward end, which can be seen in the top motor in the photograph below. All G40 motors were of the new vintage. All F25 flights were of the new vintage except for the $80^0$ flight of the Weasel.

**MOTORS FROM THE 07/15/2007 WEASEL FLIGHTS**
**THE TOP MOTOR (NEW DESIGN) WAS USED IN THE VERTICAL FLIGHT**
**THE BOTTOM MOTOR (OLD DESIGN) WAS USED IN THE $80^0$ FLIGHT**




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## COMPUTER PROGRAM

Data analysis was performed by the program, OVAA2.xls v 1.3. It is written in *Microsoft Excel VBA*™, and uses the *Excel Solver* to perform angular backtracking. It is in the public domain, and is (at this writing) available, gratis, in the downloads section of the web site www.nepra.com.

## METHOD

The Lutherlyn flights used Aerotech G40 motors. The substituted Vulcanite flight used an Aerotech G80 motor. The Weasel flights used Aerotech F25 motors. Propellant masses for these motors were obtained from www.aerotech-rocketry.com.

All flights were simulated in the *Excel VBA* program, *2DQD2.xls*, prior to launch. (This program is also in the public domain, and available for free download.) Though the simulations did not feature exact launch weights, they provided very useful estimates of where to stand down range. The simulations also revealed launch angles, motors, and delays that were realistic for the field. I settled on $75^0$ and $80^0$, and $90^0$ launches for the Lutherlyn field.

Launch rods were positioned with a protractor, using a metal beaded chain as a plumb bob. The chain does not flex the rod as a real plumb bob might. It is also easier to use when the launch angle is supposed to be perfectly vertical. The chain was prone to move in the wind and to jiggle about, however. As a result, the system proved less accurate than I had hoped. The overall accuracy was about three degrees. Naturally, launch rods flex too.

Some launch rods were measured informally by pencil marking, on the rocket, the point corresponding to the top of the rod. Others were measured formally with a tape measure.

The substituted vertical *Vulcanite* flight, and the $70^0$ *Weasel* flights carried the RDAS Compact computer and recorded 200 samples per second; the Lutherlyn flights carried an Ozark ARTS computer and recorded 100 samples per second. The flight computers were mounted on plastic sheets fashioned from flexible kitchen cutting mats. This material, which is available in several thicknesses, is stiff enough that it conforms to the inside of a body tube. In this curved state, flight accelerations will not crush it. In each rocket, it was held in place by a nose cone attached with a small screw.

Before each launch, the instrument was partially extracted with long-nosed pliers. The rod was positioned. Then rocket was then mounted, on the rod, and the instrument was turned on. The assembly was then carefully pushed back into the instrument compartment, the nose was attached, and the angle was verified. It was important to mount the rocket on the rod before activating the instrument, because the instrument calibrates its accelerometer automatically on the assumption that its position is vertical. Since the analysis program had to compensate for this incorrect calibration, it was important to keep the instrument at the intended launch angle prior to ignition.



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I paced off the down-range ejection distance predicted by the simulator and waited there, since it was apparent that the rockets would otherwise be lost. The LCO was asked to wait for relative calm before pressing the button. Air temperature was taken before and after the flight on a car dashboard thermometer.

## DATA REDUCTION METHODS

Data were analyzed from a priori conditions in the *OVAA2.xls* program in vertical and off-vertical workups. Temperature corrected and uncorrected barometric apogees were reported. Motor impulses were computed. Launch angles were also backtracked to maximize the $R^2$ statistic. Stopping criteria for this procedure were objective because the *OVAA2.xls* program performs angular backtracking without human intervention. The backtracked angles were reported and compared with the corresponding intended angles.



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# ANALYSIS



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# OVERVIEW OF DATA

## VULCANITE

| | | | |
|---|---|---|---|
| Angle | 75 | 80 | 90 |
| Backtracked Angle | 76.45585 | 79.96541 | 90 |
| | | | |
| OVAA | 974.4767 | 1232.636 | 1106.077 |
| Vertical Inertial Altitude | 1166.129 | 1357.494 | 1106.077 |
| | | | |
| Corrected Barometric Altitude | 996.4528 | 1210.971 | 1132.466 |
| %Error OVAA Corrected Baro | -2.20543 | 1.789082 | -2.33019 |
| %Error Vertical / Corrected Baro | 17.02799 | 12.09965 | -2.33019 |
| | | | |
| Uncorrected Barometric Altitude | 964.8294 | 1172.539 | 1139.056 |
| %Error OVAA / Uncorrected Baro | 0.999896 | 5.125335 | -2.89523 |
| %Error Vertical / Uncorrected Baro | 20.86371 | 15.77385 | -2.89523 |

## STRETCH INITIATOR

| | | | |
|---|---|---|---|
| Angle | 75 | 80 | 90 |
| Backtracked Angle | 74.72732 | 77.11239 | 82.92605 |
| | | | |
| OVAA | 910.0242 | 1105.594 | 1027.063 |
| Vertical Inertial Altitude | 1064.443 | 1184.004 | 1027.063 |
| | | | |
| Corrected Barometric Altitude | 904.4439 | 1036.201 | 969.7524 |
| %Error OVAA / Corrected Baro | 0.616988 | 6.696879 | 5.909821 |
| %Error Vertical / Corrected Baro | 17.69028 | 14.26391 | 5.909821 |
| | | | |
| Uncorrected Barometric Altitude | 887.336 | 1016.601 | 938.9764 |
| %Error OVAA Uncorrected Baro | 2.556892 | 8.754004 | 9.381136 |
| %Error Vertical / Uncorrected Baro | 19.95936 | 16.46693 | 9.381136 |

## WEASEL

| | |
|---|---|
| Angle | 70 |
| Backtracked Angle | 67.22628 |
| | |
| OVAA | 1205.613 |
| Vertical Inertial Altitude | 1385.773 |
| | |
| Corrected Barometric Altitude | 1196.856 |
| %Error OVAA / Corrected Baro | 0.731667 |
| %Error Vertical / Corrected Baro | 15.78444 |
| | |
| Uncorrected Barometric Altitude | 1172 |
| %Error OVAA / Uncorrected Baro | 2.868003 |
| %Error Vertical / Uncorrected Baro | 18.24002 |



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## WEASEL

| | | | |
|---|---|---|---|
| Angle | 70 | 80 | 90 |
| Backtracked Angle | 70.98465287 | 77.51857184 | 86.50715 |
| | | | |
| OVAA | 1251.2 | 1820.03 | 1625.6 |
| Vertical Inertial Altitude | 1630.1 | 1945.09 | 1625.6 |
| | | | |
| Corrected Barometric Altitude | 1275.55 | 1752.13 | 1575.25 |
| %Error OVAA / Corrected Baro | -1.90898 | 3.875283 | 3.196318 |
| %Error Vertical / Corrected Baro | 27.79585 | 11.01288 | 3.196318 |
| | | | |
| Uncorrected Barometric Altitude | 1293 | 1680.84 | 1519.59 |
| %Error OVAA Uncorrected Baro | -3.23279 | 8.280979 | 6.976224 |
| %Error Vertical / Uncorrected Baro | 26.07115 | 15.72131 | 6.976224 |

## SUMMARY STATISTICS

| | *OVAA* | *Corrected* | *Raw* | *Vertical* |
|---|---|---|---|---|
| Mean | 1225.831493 | 1205.007985 | 1178.476441 | 1330.820812 |
| Standard Error | 90.99498495 | 85.92271237 | 81.4755474 | 90.90606536 |
| Median | 1155.845 | 1164.661 | 1155.528 | 1270.749 |
| Standard Deviation | 287.7514081 | 271.7114738 | 257.6483034 | 287.4702197 |
| Sample Variance | 82800.87287 | 73827.12501 | 66382.64824 | 82639.12719 |
| Skewness | 1.253347246 | 1.110916002 | 0.937942664 | 1.129600475 |
| Range | 910.0105394 | 847.6908584 | 793.5039488 | 918.0241281 |
| Minimum | 910.0242 | 904.4439 | 887.336 | 1027.063 |
| Maximum | 1820.034739 | 1752.134758 | 1680.839949 | 1945.087128 |
| Count | 10 | 10 | 10 | 10 |

Inertial altitude curves are quite smooth as a result of being integrated twice, while barometric curves are subject to individual outlying values and to stair stepping from low precision. Both of these effects conspire to make barometric ascent times slightly shorter, on average, than inertial times are (Appendix B). Barometric altitude at burnout is subject to aerodynamic effects that attend high speeds. Inertial values are likely better.

The mean OVAA altitude was about 20 feet greater then the mean temperature-corrected barometric altitude, which was about 25 feet greater than the raw barometric altitude. Interestingly, the median raw barometric altitude was almost exactly the mean OVAA altitude, but the corrected median was only about 9 feet different.

Results for the Initiator were slightly less accurate than results for the other rockets. Aerodynamic influence is visible in the altitude/time graphs in Appendix B. This may be because the flights occurred earlier in the day when the weather was windier. It may be



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because the stretched configuration is less stable than the others and requires more oscillation.

In general, OVAA errors from temperature-corrected altitudes were greatest at $80^0$, next greatest in vertical launches, and smallest at $70^0$ or $75^0$. Given the small data set, no conclusion can be drawn. If this is a real pattern, then perhaps AOA effects from the faster-turning trajectory tangents at steeper angles are involved. OVAA's absolute error from barometric altitude certainly did not increase monotonically with launch angle.

The first flight of the Weasel shows evident altimeter delay because of small static ports (Appendix B). The advertised six second delay was only about four seconds. Quoted altitudes were from analysis with the data set truncated at ejection, but the altitude-time curves are depicted both ways. There are enough data here to find drag coefficients and thrust curves, so that the data can be extended in a simulator to reveal performance with a full delay.

Interestingly, all vertical flights resulted in lower maximum altitudes than the corresponding $80^o$ flights, and this was the case for barometric altitudes as well as inertial altitudes. The reasons were three slightly odd circumstances. (1) The vertical *Vulcanite* launch hung up on the launch rod as previously described, and squandered impulse in the process. (2)It turns out that the motor in the $90^o$ *Initiator* burned somewhat long. The configuration was such that the extra burning time resulted in lost altitude. (3) The $80^0$ launch of the Weasel used an F25 motor of older and more powerful vintage (See motor discussion below) and hence went uncharacteristically far – outstripping the corresponding vertical launch.

A curious result is that the $70^0$*Weasel* launches were rather good, even though they took place in very windy weather by rocket standards. Launches were into the wind, rod speed was fast, and launch angles were very off-vertical. This means that velocity from lift-off to nose-over was reasonably high, and the proportional influence of wind over the trajectory was that much less. Launching into the wind makes the rocket horizontal at nose-over just as off-vertical launching does, and the total angular turning from lift to nose-over was only $70^0$. Thus, not only is the wind's influence less, the wind is making the rocket act not terribly differently than it otherwise would.

**For these reasons, it may be that a significantly off-vertical launch into the wind tends to yield more accurate results than a vertical launch under the same conditions.** Further investigation of this point is suggested.

## OVAA ALTITUDES AND BAROMETRIC ALTITUDES AND TEMPERATURE

Barometric altitudes array nicely against inertial altitudes, as illustrated in the following graph. The dots with the arrows are from vertical flights.



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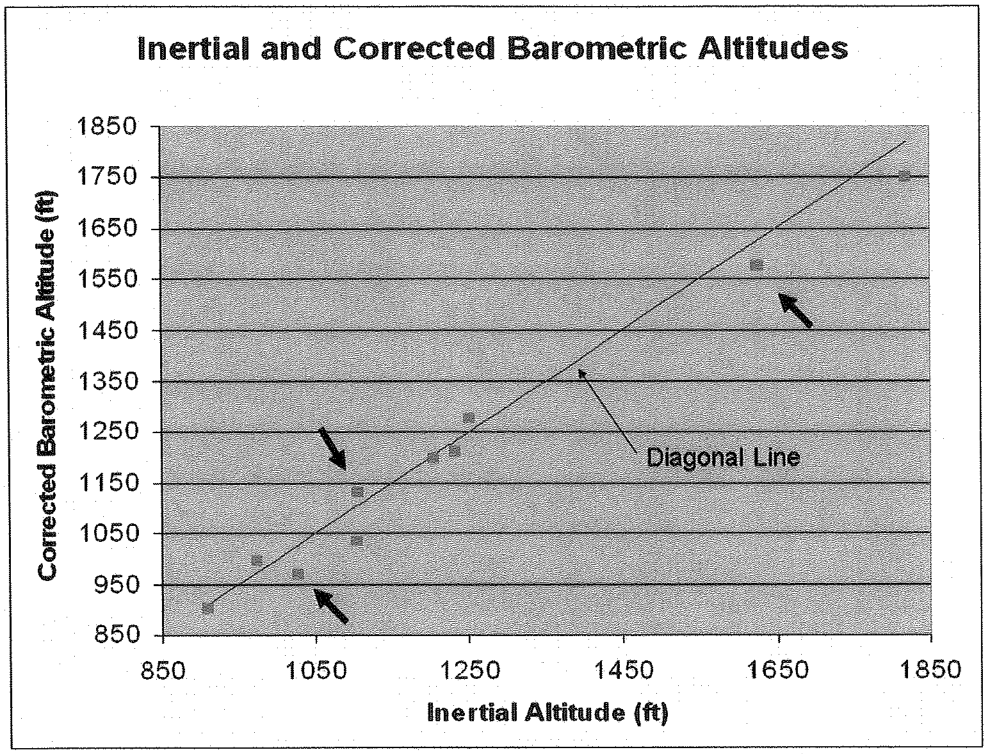
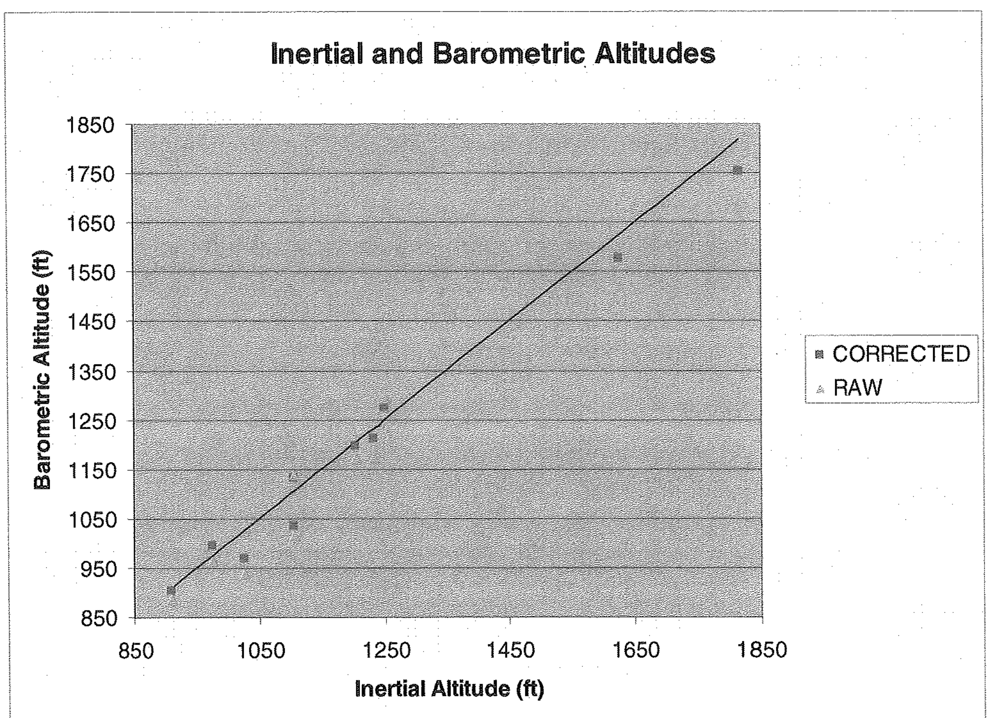



<u>The line in the middle is diagonal, and is not fitted specially to the points about it</u>. The barometric altitudes in the graph above are all temperature-corrected. Raw barometric altitudes are added as triangular points in the graph below.



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Note that the corrected altitudes are visibly closer to the diagonal line than the uncorrected altitudes are. This was true at every point but 1, the vulcanite $75^0$ launch, where both errors were small. Differences between raw and corrected altitudes are greater when temperatures are more different from $59^0$ F. They also increase at higher altitudes, because the correction is multiplicative. The temperature range in these graphs is from $52^0$ F and $81^0$ F, and the greatest altitude is something less than 1850 feet. These ranges are relatively modest, and the difference is evident. Corrections to, say, a 5000-foot flight at $94^0$ would be considerable.

## REGRESSION ANALYSIS

It seems natural to do a regression analysis on these data. Here are the results of regression inertial altitude on corrected and raw barometric altitudes.

| | *Corrected* | *Raw* |
|---|---|---|
| **Correlation Coefficient** | 0.992224757 | 0.982094129 |
| **$R^2$** | 0.984509968 | 0.964508879 |
| **Standard Error** | 37.985677 | 57.49811117 |
| **Observations** | 10 | 10 |

| | Coefficients | Std Error | t Stat | P-value |
|---|---|---|---|---|
| Intercept | -40.38936521 | 57.42439314 | -0.703349 | 0.5017916 |
| **Corrected** | 1.050798728 | 0.046600507 | 22.54908 | 1.5841E-08 |

| | Coefficients | Std Error | t Stat | P-value |
|---|---|---|---|---|
| Intercept | -66.76862316 | 89.5306933 | -0.745762 | 0.4771492 |
| Raw | 1.096840014 | 0.074388369 | 14.74478 | 4.4015E-07 |

Slopes are both within two standard deviations of unity. The intercepts are not statistically significant, and we can eliminate them.

| | *Slopes* | *Std Error* | *t Stat* | *P-value* | *Low 95%* | *Hi 95%* |
|---|---|---|---|---|---|---|
| **Corrected** | 1.018747 | 0.009470 | 107.5721 | 2.631E-15 | 0.997324 | 1.040171 |
| **Raw** | 1.042520 | 0.014730 | 70.77494 | 1.13E-13 | 1.009198 | 1.075842 |

The slope of the *Corrected* altitudes is within the 95% confidence interval. That of the uncorrected altitudes is not.

It must be revealed that regression analysis, as normally applied, assumes that the values of independent variables are known with absolute certainty. To the extent that barometric altitudes are uncertain, slopes are apt to be biased. (See Reference 11)

Even so, the analysis substantiates a strong linear relationship between OVAA and barometric altitudes, and suggests that the slope in the relationship is close to unity. This test concerns altitude alone, and further research is required to test horizontal distance.

## EFFECT OF TEMPERATURE CORRECTION ON CLOSURE: Binomial Test

Isolation of the effect of temperature correction is complicated by the scarcity of data and the limited temperature range. The most straightforward way to test it is to count the total number of flights and the number of flights where corrected barometric temperature was closer to inertial altitude. We can then use the binomial distribution to estimate the chance of an outcome at least this extreme if temperature correction made no difference. That is, we assume a null hypothesis that the chance of improvement from temperature correction is 50%.

| | |
|---|---|
| **Number of Flights:** | **10** |
| **Number of improvements:** | **9** |
| **Chance of result or more extreme under null hypothesis:** | **.0107** |

Given the result, we can reject the null hypothesis that correction has no positive effect.

We can also perform **a paired t-test** to test the null hypothesis that the real mean difference between inertial and barometric altitudes is zero. This test is more powerful than the pooled t-test implicit in the regression analysis, because it exploits the unique relationship between data values pertaining to the same flight.

If temperature correction is productive and OVAA is a good estimator, then we should hope that

1) OVAA altitudes ARE NOT distinguishable from corrected barometric altitudes
2) OVAA altitudes ARE distinguishable from raw barometric altitudes.

This is, indeed, what happens:

**PAIRED t-TEST RESULTS**

| *CANNOT REJECT!* | *OVAA* | *Corrected* |
|---|---|---|
| **Mean** | 1225.831493 | 1205.007985 |
| **Variance** | 82800.87287 | 73827.12501 |
| **Observations** | 10 | 10 |
| **Hypothesized Mean Difference** | 0 | |
| **Degrees of Freedom** | 9 | |
| **t Stat** | 1.715685925 | |
| **P(T<=t) two-tail** | 0.12035646 | |
| **t Critical two-tail** | 2.262157158 | |



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| *REJECT!* | *OVAA* | *Raw* |
|---|---|---|
| **Mean** | 1225.831493 | 1178.476441 |
| **Variance** | 82800.87287 | 66382.64824 |
| **Observations** | 10 | 10 |
| **Hypothesized Mean Difference** | 0 | |
| **Degrees of Freedom** | 9 | |
| **t Stat** | 2.509378712 | |
| **P(T<=t) two-tail** | 0.03334473 | |
| **t Critical two-tail** | 2.262157158 | |

That said, there is little doubt that with more data, OVAA could be shown to be distinguishable from both barometric altitudes. In a way, that is a trivial statement because every statistician knows enough data will inevitably make any two quantities distinguishable. In this case, though, the *underlying principle* contains a small biased error term that is nonzero, and the utility of single-axis accelerometer analysis rests on squared error, rather than on unbiasedness.

On the other hand, it is remarkable that the paired t-test shows a significant difference between OVAA and uncorrected altitudes in this small data set. If we concentrate on that result, the indistinguishability of OVAA from corrected altitudes serves as an anchor, demonstrating that OVAA values are not just random numbers, which would also present as values different from uncorrected altitudes.

The previous binomial distribution test of improvement is very strong, by contrast, and, being based on a significant difference (rather than an indistinguishable difference), it is likely stand the test of more data.

## COMPARISON OF OVAA AND VERTICAL ANALYSIS

Rockets are frequently launched in slightly off-vertical trajectories with accelerometers aboard, with the idea that the small off-vertical angle will not matter too much in a vertical estimation of altitude. An important finding that emerges immediately from the *Tabulated Data* in this paper is that, in every case, OVAA analysis of off-vertical trajectories yields altitudes that are closer to barometric altitudes than vertical inertial altitudes are, and the improvement is very great. The differences in error increase with deviation from vertical orientation, as would be expected. Of course, in the vertical case, the two methods are identical, and there is no difference at all.

## CLOSURE AND OVAA v FACE VALUE VERTICAL ANALYSIS

The shaded areas in the ***Tabulated Results*** highlight the comparisons of OVAA with temperature adjusted barometric altitude and vertical inertial altitude with unadjusted temperature. These are the comparisons that would form the basis of closure criteria. The comparison of OVAA with adjusted temperature is best in every case, suggesting that IBC is more likely effective with OVAA and temperature correction.



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It would seem that, if off vertical angles, even small ones, are to be used, inertial/barometric closure is more likely to be useful when inertial altitudes are derived by means of OVAA and when temperature corrected barometric altitudes are used.

## ANGULAR BACKTRACKING RESULTS

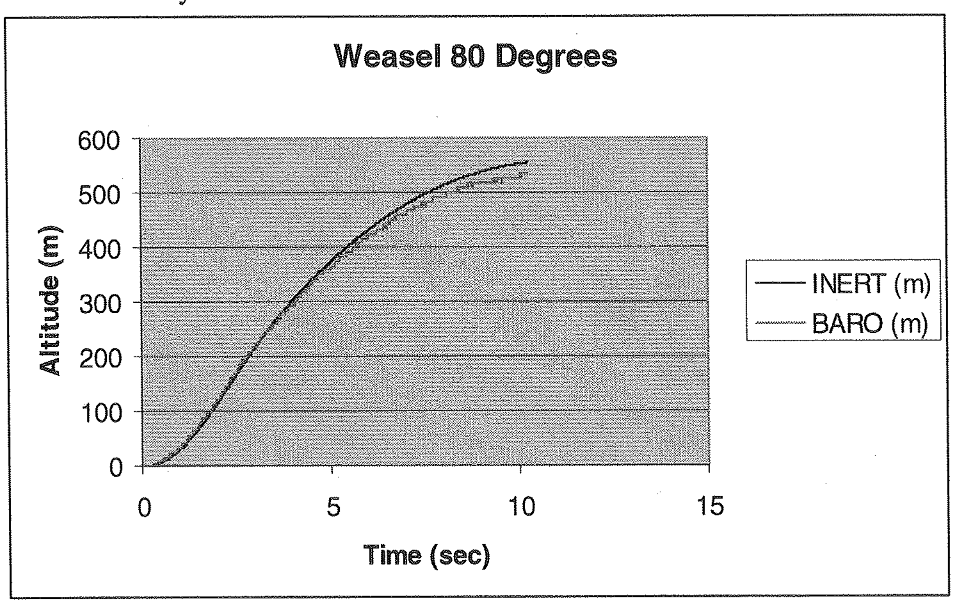
Omitting the flight that tipped off and is examined below, all backtracked angles except two were within three degrees of the intended launch angle. Both flights that did not close within three degrees were vertical, and that includes the tipped off flight.

One flight, the substituted G80 Vulcanite flight, had exactly the same backtracked angle as intended angle, and this was also a vertical flight. This result is not wildly coincidental. The barometric altitude was very slightly higher than the OVAA altitude, and the most vertical angle is $90^0$. Therefore, the exact correspondence reflects a right wall, rather than an absolutely perfect match.

## BACKTRACKING FOR ANGULAR REFINEMENT

My angle setting technique was frankly poor, and subject to the equipment available at a particular launch site. Between a plumb chain blowing in the wind, and some launchers that held their launch angle with the help of prayers, I had much less control over this important variable than I might have wished, and my accuracy was only about $3^0$-$4^0$.

We might entertain the notion that angular backtracking can be used to refine launch angles after the fact. Here are altitude time curves for two Weasel flights that have been treated this way.



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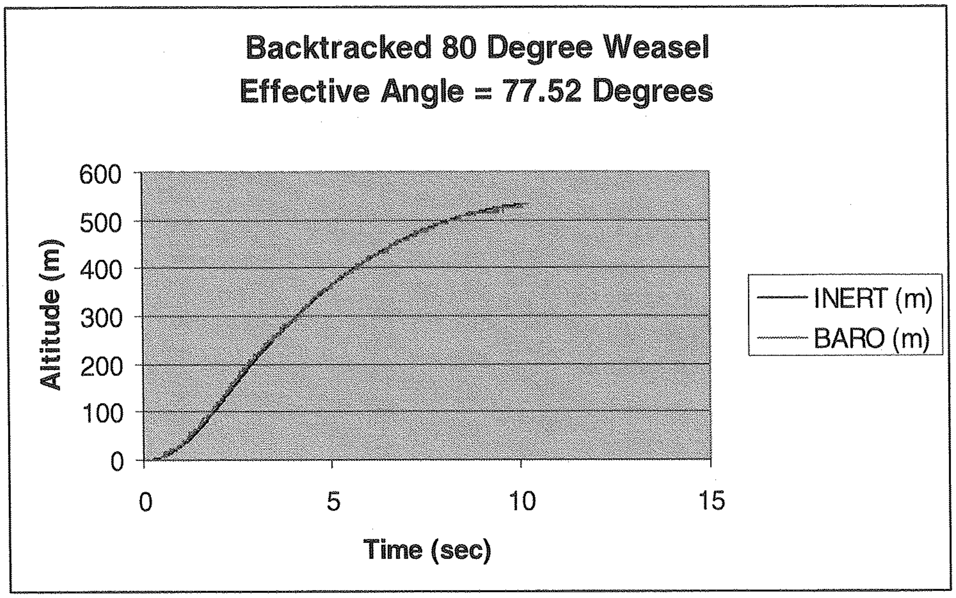
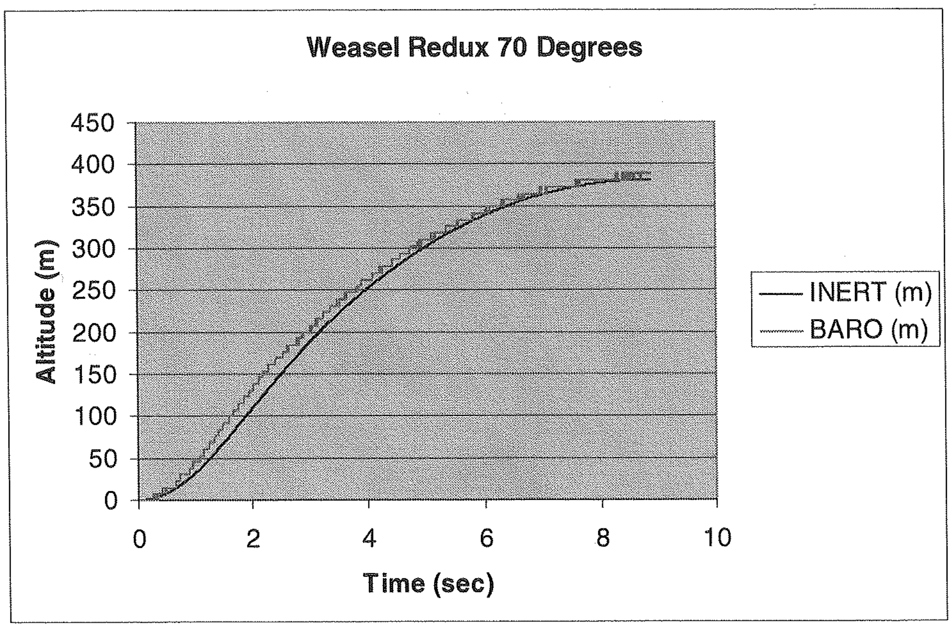
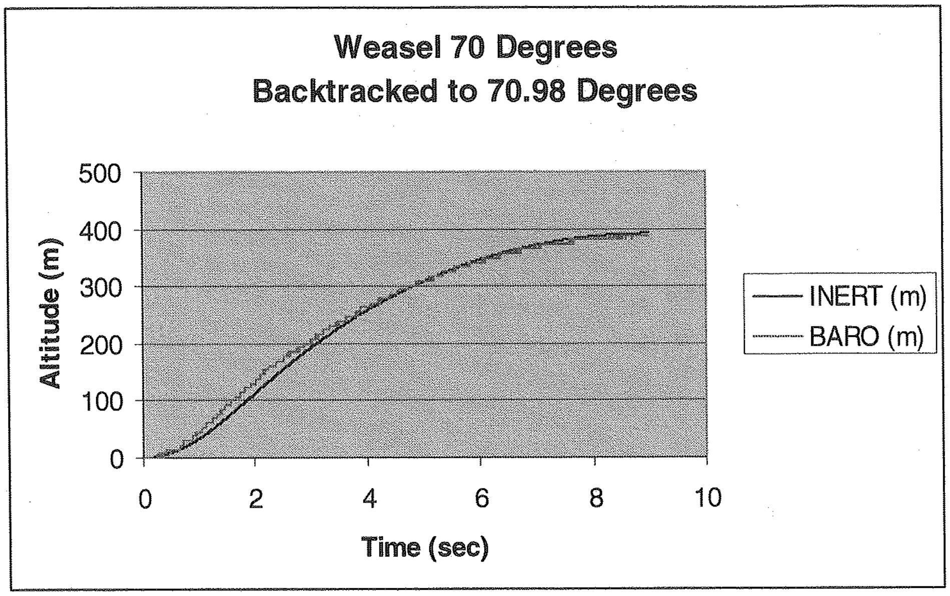

Backtracked 80 Degree Weasel
Effective Angle = 77.52 Degrees
600
500
400
300
200
100
0
Altitude (m)
0
5
10
15
Time (sec)
INERT (m)
BARO (m)


Weasel Redux 70 Degrees
450
400
350
300
250
200
150
100
50
0
Altitude (m)
0
2
4
6
8
10
Time (sec)
INERT (m)
BARO (m)




Again, the conservative interpretation is that the actual fit is no worse than the fit at the intended angle and no better than the fit at the backtracked angle. The utility of backtracking to refine launch angle is a credible range for velocity data, thrust and drag coefficient data. The procedure, alas, is not provable.

## CLOSURE AND SELF-CALIBRATING INSTRUMENTS

The shaded areas in the tabulated results highlight comparisons that would form the basis of closure criteria. Here are some ground rules:

1) The first error column has errors from temperature-corrected barometric altitudes.
2) The second error column has comparisons with uncorrected barometric altitudes.
3) The term, *Calibrated*, refers to inertial data that have been analyzed, taking the off-vertical self-calibration angle into account.
4) The term, *Uncalibrated*, denotes inertial altitudes derived from self-calibrating accelerometer data taken at face value.
5) OVAA denotes off vertical analysis done on calibrated data.

The examples below are workups of all the $80^0$ launches. Let us consider the launch of the Vulcanite.

1) This paper proposes closure criteria involving intended angle OVAA and temperature-corrected altitude. The error in that comparison is 1.8%.
2) A hobby rocket enthusiast, using face value data and believing s/he can get away with an $80^0$ launch, encounters almost four times that error or 15.8%.
3) Had that same enthusiast made the same decisions before self-calibrating accelerometers, the error would have been only about half as large, or 8.3%.

Barring backtracked OVAA, the comparison of OVAA with adjusted temperature is best, suggesting that IBC is more likely effective with OVAA and temperature correction.

### VULCANITE $80^0$

| Altitude (ft) | Method | %Error Corrected | %Error Uncorrected |
|---|---|---|---|
| 1173 | Barometric Uncorrected | -3.2 | 0 |
| 1211 | Barometric Corrected | 0 | 3.3 |
| 1232 | Backtracked OVAA | 1.7 | 5.1 |
| 1233 | Intended Angle OVAA | 1.8 | 5.1 |
| 1311 | Vertical Calibrated | 8.3 | 11.8 |
| 1357 | Vertical Uncalibrated | 12.1 | 15.8 |

### STRETCH INITIATOR $80^0$

| Altitude (ft) | Method | %Error Corrected | %Error Uncorrected |
|---|---|---|---|
| 1017 | Barometric Uncorrected | -1.9 | 0 |
| 1036 | Barometric Corrected | 0 | 1.9 |
| 1054. | Backtracked OVAA | 1.8 | 3.7 |
| 1106 | Intended Angle OVAA | 6.7 | 8.8 |
| 1149 | Vertical Calibrated | 10.9 | 13.0 |
| 1184 | Vertical Uncalibrated | 14.3 | 16.5 |



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### WEASEL $80^0$

| Altitude (ft) | Method | %Error Corrected | %Error Uncorrected |
|---|---|---|---|
| 1681 | Barometric Uncorrected | -4.1 | 0.0 |
| 1752 | Barometric Corrected | 0.0 | 4.2 |
| 1751 | Backtracked OVAA | -0.1 | 4.2 |
| 1820 | Intended Angle OVAA | 3.9 | 8.3 |
| 1890 | Vertical Calibrated | 7.9 | 12.4 |
| 1945 | Vertical Uncalibrated | 11.0 | 15.7 |

A more important question is What is the effect of self-calibration on flights where the intended angle is inaccurately set, when OVAA is used? The issue deserves attention because anyone who has tried to set an angle accurately can tell you how frustrating the endeavor can be.

We shall suppose, for the sake of argument, that backtracked angles are the true launch angles. Additional error must accrue because the accelerometers are calibrated at the true (unknown) angles, and are assumed to be calibrated on the intended angle. How big would this additional error be? Here are the results for two flights.

### WEASEL $80^0$

| Altitude (ft) | Method | %Error Corrected |
|---|---|---|
| 1752 | Barometric Corrected | 0.0 |
| 1751 | Backtracked OVAA | -0.1 |
| 1820 | Intended Angle OVAA | 3.9 |
| 1790 | Recalibrated OVAA | 2.2 |

### STRETCH INITIATOR $80^0$

| Altitude (ft) | Method | %Error Corrected |
|---|---|---|
| 1036 | Barometric Corrected | 0 |
| 1054. | Backtracked OVAA | 1.8 |
| 1106 | Intended Angle OVAA | 6.7 |
| 1083 | Recalibrated OVAA | 4.5 |

Recalibration accounted for about 44% of the Weasel error and about 33% of the Initiator error. It must be admitted that even a formal calibration procedure is subject to error, since it requires a vertical reference and a horizontal reference. These are relatively easy to arrange. (A vertical reference can be had by suspending the instrument in a wind-shielded area; a horizontal reference may be made with a spirit level.) It is another matter to use someone else's launch equipment and to set an off-vertical angle in the wind with a protractor. Bottom line: Self-calibrating accelerometers are much less forgiving of errors in launch angles than are manually calibrated instruments.

It would seem that self-calibrating accelerometers, which are designed to make flying easier, can contribute greatly to lack of closure in slightly off-vertical flights. No wonder that the sellers of some of these instruments do not display inertial altitude!

## FLIGHT IMPULSE COMPUTATION

Flight impulses were computed, averaged, and compared with impulse values from Aerotech's 2005-2006 catalog (latest available on line at this writing), Tripoli and NAR. Here are the results of that analysis.

| MOTOR | NAR IMPULSE | MANUFACTURER'S IMPULSE (ns) | FLIGHT IMPULSE (ns) |
|---|---|---|---|
| G40 | 97.1 | 100 | 95.2-96.0 |
| G80 | 94 | 100 | <Corrupted by Launch Rod> |
| F25 | 77(*) | 73 | 70.4 (new) and 75.2 (old) |

The high value in the G40 flight impulse range includes data from the remediated flight (See below); the low value does not. These motors have been recently redesigned, and their impulse has been reduced. The G40 motor had been rated as 120 ns. In May of 2007, NAR S&T (R110) re-rated it at the above value. A similar redesign befell the F25, and it is reflected in the manufacturer's impulse rating. I do not see it reflected in NAR statistics as yet, but the flight value is close to the manufacturer's figure.

## REMEDIATION OF BAD DATA FROM LAUNCHER TIP-OFF

In addition to the flights presented above, one *Vulcanite* flight, which was intended to be vertical, tipped off the launch rod, fish tailed briefly, and quickly reacquired stability in an off-vertical direction. Stability was established within about 70 feet of the ground, so it is reasonable to consider angular backtracking as a method of data remediation. There is no way to verify the validity of these results. They are presented as estimates.

In this particular case, the underlying assumption is that the calibration angle was $90^0$, that the rocket tipped off just at launch, and thereafter the flight was stable. The program was informed of the calibration angle and asked to vary the launch angle to maximize the $R^2$ statistic.



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# VULCANITE TIPPED OFF (UNCORRECTED AT 90$^0$)

| | |
|---|---|
| TEMPERATURE C | 25.55555556 |
| LAUNCH SITE ALTITUDE (m) | 253 |
| | |
| DIAMETER (cm) | 5.7404 |
| LAUNCH MASS (g) | 944 |
| PROPELLANT MASS (g) | 53.8 |
| LAUNCH ROD LENGTH (cm) | 134.62 |
| LAUNCH ANGLE (Degrees) | 90 |

| | | English |
|---|---|---|
| RSquare | 0.835413618 | |
| Barometric Apogee (m) | 288.8057185 | 947.525 |
| Inertial Alt at Barometric Apogee (m) | 345.5510102 | 1133.7 |
| Distance at Barometric Apogee (m) | 1.15813E-12 | 3.8E-12 |
| Barometric Ascent Time (sec) | 7.5 | |
| | | |
| Inertial Apogee (m) | 358.1069692 | 1174.89 |
| Speed at Inertial Apogee (m/sec) | 0.02274975 | 0.07464 |
| Throw Distance at Inertial Apogee (m) | 1.39529E-12 | 4.6E-12 |
| Inertial Ascent Time (sec) | 9.07 | |
| | | |
| Baro Altitude at Burnout (m) | 135.1250014 | 443.324 |
| Inertial Altitude at Burnout (m) | 145.8722222 | 478.583 |
| Velocity at Burnout (m/sec) | 71.90586675 | 235.912 |
| | | |
| Maximum Velocity (m/sec) | 76.82066675 | 252.036 |
| Maximum Mach Number | 0.221884376 | |
| | | |
| Max Acceleration (m/sec^2) | 55.43 | 181.857 |
| Min Acceleration (m/sec^2) | -14.78 | -48.491 |
| | | |
| Burning Time (sec) | 2.8 | |
| Impulse (ns) | 98.66659341 | 438.74 |
| | | |
| Specific Impuse (sec) | 187.011013 | |
| Effective Exhaust Velocity (m/sec) | 1789.3507 | 5870.57 |



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# VULCANITE TIPPED OFF (AT EFFECTIVE ANGLE)

| | |
|---|---|
| TEMPERATURE C | 25.55555556 |
| LAUNCH SITE ALTITUDE (m) | 253 |
| | |
| DIAMETER (cm) | 5.7404 |
| LAUNCH MASS (g) | 944 |
| PROPELLANT MASS (g) | 53.8 |
| LAUNCH ROD LENGTH (cm) | 134.62 |
| LAUNCH ANGLE (Degrees) | 68.38182019 |

| | | English |
|---|---|---|
| RSquare | 0.996136898 | |
| Barometric Apogee (m) | 288.8057185 | 947.525 |
| Inertial Alt at Barometric Apogee (m) | 287.8042151 | 944.24 |
| Distance at Barometric Apogee (m) | 260.9532426 | 856.146 |
| Barometric Ascent Time (sec) | 7.5 | |
| | | |
| Inertial Apogee (m) | 291.7034556 | 957.032 |
| Speed at Inertial Apogee (m/sec) | 36.13955136 | 118.568 |
| Throw Distance at Inertial Apogee (m) | 293.2817977 | 962.211 |
| Inertial Ascent Time (sec) | 8.39 | |
| | | |
| Baro Altitude at Burnout (m) | 135.1250014 | 443.324 |
| Inertial Altitude at Burnout (m) | 128.612322 | 421.956 |
| Velocity at Burnout (m/sec) | 75.3866115 | 247.331 |
| | | |
| Maximum Velocity (m/sec) | 79.39061106 | 260.468 |
| Maximum Mach Number | 0.229285494 | |
| | | |
| Max Acceleration (m/sec^2) | 56.3751542 | 184.958 |
| Min Acceleration (m/sec^2) | -14.1651597 | -46.474 |
| | | |
| Burning Time (sec) | 2.8 | |
| Impulse (ns) | 97.65848204 | 434.26 |
| | | |
| Specific Impuse (sec) | 185.1002555 | |
| Effective Exhaust Velocity (m/sec) | 1771.080972 | 5810.63 |



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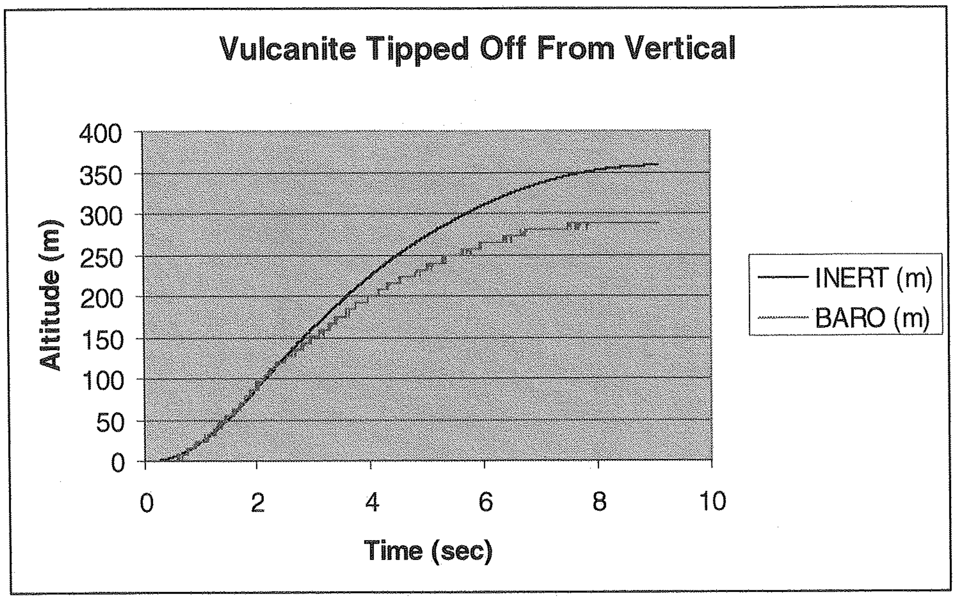


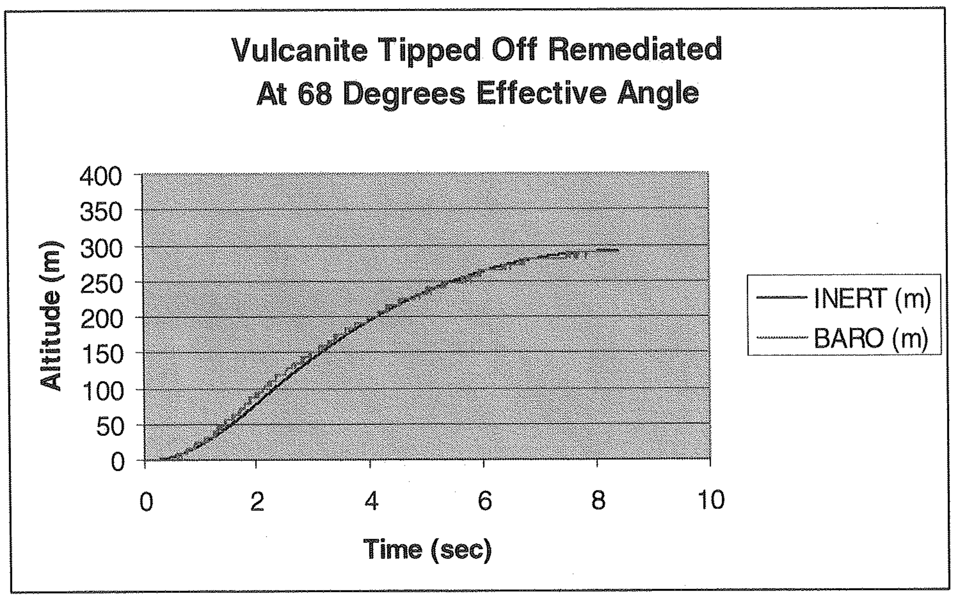




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## RESEARCH QUESTIONS REVISITED

**How well does OVAA work in practice?**
Evidently well. All the flights closed within 7%, except for the one flight that tipped off badly. That failure was legitimate. OVAA altitudes array against barometric altitudes such that they straddle a horizontal line. Fitted sloes are near unity.

**How do backtracked launch angles compare with intended launch angles?**
Pretty well. All the off-vertical launch angles were within $3.5^0$ of intended launch angle. That at least in part reflects the difficulty in accurately setting launch angle with found equipment.

**How does OVAA analysis compare with vertical analysis of off-vertical flights?**
OVAA is a very great deal better.

**Does temperature correction of altimeter data contribute to inertial/barometric closure?**
Yes. Statistical tests reveal a substantial benefit, even in this small data set over a limited temperature range.

**In vertical analysis, what is the effect, on data from off-vertical trajectories, of not compensating for self-calibrating accelerometers?**
Incorrect calibration from self-calibrating accelerometers can account for a substantial portion of total altitude error.

**In OVAA analysis, what is the effect, on data from flights with launch angle errors, of accelerometer self-calibration?**
Hypothetical experiments that accept backtracked angle as true launch angle suggest that self-calibration can contribute a substantial portion of error.

**How accurately can motor impulses be determined from flight data?**
This question is complicated by the recent redesign of the motors used in these experiments. The average flight impulse for the G40 was within about 2% of the new NAR impulse. That for the F25 was pretty close to the manufacturer's impulse. The NAR impulse, which was not adjusted for redesign, was substantially higher than the manufacturer's velue.

**Can launcher tip-off be remediated through angular backtracking?**
Impossible to say with certainty. Experimental data look good, but there is no way to objectively verify them.



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## CONCLUSIONS

The major conclusion of this paper is:

******

**Single-axis accelerometers do NOT constrain us to vertical trajectories, as once believed. Rather, they constrain us to substantially BALLISTIC trajectories – whether vertical or off-vertical.**

******

It follows that lift and wind can undermine the usefulness of single axis accelerometer data.

Inertial/barometric closure can be greatly improved by OVAA, temperature correction of altimeter data, and by compensation for self-calibrating accelerometers in off-vertical launches. It follows that results from standard flight computer software can not always be taken at face value.

The experimental results of this paper demonstrate that OVAA is a very natural and correct extension to conventional analysis that has been previously overlooked, undoubtedly because accelerometer data were thought to be attitude-dependent. Once gravitational oblivion is demonstrated, apparent obstacles to analysis of off-vertical inertial trajectories disappear.

## FURTHER RESEARCH

This paper examines OVAA altitude, since barometric verification is readily available. Throw distance is not tested, and it should be.

Also, it is possible that filtering could reduce bias from stability oscillations. Filters constitute a big topic, because of their underlying assumptions, and their behavior around singularities and discontinuous derivatives, which may occur in acceleration data. It is also possible that they will improve primary trajectory statistics by removing small but *real* lumps in the path, and under-estimate impulse by the same mechanism.

The effect of wind should be investigated. As speculated in the results discussion, it may be that significantly off-vertical flights launched into the wind result in more accurate trajectory numbers than vertical flights would yield under the same conditions.



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It may be that the accuracy of OVAA analysis has something to do with the steepness or shallowness of trajectory. Perhaps more off-vertical angles yield slightly more accurate data, because the tangents to its trajectory, and hence the angles of a stable rocket very at a slow rate. This should be investigated.

Finally, there may be an effect of stability margin on accuracy of accelerometer analysis in general, as stability relates to oscillations and angle of attack.

## BUDGET

| | |
|---|---|
| Rockets | $200.00 |
| Motors | $275.00 |
| Cheap Motel Room | $40.00 |
| Flight Computers | $610.00 |
| Printing (Estimated) | $30.00 |
| | |
| Total | $1,155.00 |



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# APPENDIX A
# RECOMMENDED DATA LIST

**REQUIRED**

Launch Mass – Actually measured before launch. Rocket design program output doesn't count! Weights read off a kit box don't count! (Used in impulse and Isp computations)

Broadest diameter (Used in $C_d$ computations)

Propellant mass (Used in impulse and Isp computations)

Launch angle (Used for everything)

Launch Rod Length (Used for everything in off-vertical launches)

Temperature of the day (Used in barometric altitude computations)

Launch site elevation (Used in $C_d$ computations)

Description of any launch anomalies (Qualitatively helpful)

**GOOD TO HAVE**

Approximate wind speed (Qualitatively helpful)

Description of wind angle with intended launch plane (Qualitatively helpful)

Barometric Pressure (If convenient)



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# APPENDIX B: RAW DATA



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# VULCANITE 75$^0$

| | | |
|---|---|---|
| TEMPERATURE C | 24.44444444 | |
| LAUNCH SITE ALTITUDE (m) | 396 | |
| | | |
| DIAMETER (cm) | 5.7404 | |
| LAUNCH MASS (g) | 944 | |
| PROPELLANT MASS (g) | 53.8 | |
| LAUNCH ROD LENGTH (cm) | 134.62 | |
| LAUNCH ANGLE (Degrees) | 75 | |
| | | **English** |
| **RSquare** | **0.991013509** | |
| **Barometric Apogee (m)** | **303.7188069** | **996.453** |
| **Inertial Alt at Barometric Apogee (m)** | **293.8794419** | **964.171** |
| **Throw Distance at Barometric Apogee (m)** | **186.5737099** | **612.118** |
| **Barometric Ascent Time (sec)** | **7.71** | |
| | | |
| **Inertial Apogee (m)** | **297.0204929** | **974.477** |
| **Speed at Inertial Apogee (m/sec)** | **25.66141717** | **84.191** |
| **Distance at Inertial Apogee (m)** | **207.1175247** | **679.519** |
| **Inertial Ascent Time (sec)** | **8.51** | |
| | | |
| **Baro Altitude at Burnout (m)** | **134.4881088** | **441.234** |
| **Inertial Altitude at Burnout (m)** | **122.4785475** | **401.833** |
| **Velocity at Burnout (m/sec)** | **70.79448498** | **232.265** |
| | | |
| **Maximum Velocity (m/sec)** | **73.74169217** | **241.935** |
| **Maximum Mach Number** | **0.213374598** | |
| | | |
| **Max Acceleration (m/sec^2)** | **51.28980597** | **168.274** |
| **Min Acceleration (m/sec^2)** | **-13.1626354** | **-43.185** |
| | | |
| **Burning Time (sec)** | **2.72** | |
| **Impulse (ns)** | **92.76805268** | **412.514** |
| | | |
| **Specific Impuse (sec)** | **175.8310174** | |
| **Effective Exhaust Velocity (m/sec)** | **1682.390848** | **5519.66** |



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# VULCANITE $80^0$

| | |
|---|---|
| TEMPERATURE C | 24.44444444 |
| LAUNCH SITE ALTITUDE (m) | 396 |
| | |
| DIAMETER (cm) | 5.7404 |
| LAUNCH MASS (g) | 943 |
| PROPELLANT MASS (g) | 53.8 |
| LAUNCH ROD LENGTH (cm) | 134.62 |
| LAUNCH ANGLE (Degrees) | 80 |

| | | English |
|---|---|---|
| RSquare | 0.996144328 | |
| Barometric Apogee (m) | 369.1038643 | 1210.97 |
| Inertial Alt at Barometric Apogee (m) | 369.7037373 | 1212.94 |
| Distance at Barometric Apogee (m) | 171.8839474 | 563.924 |
| Barometric Ascent Time (sec) | 8.72 | |
| | | |
| Inertial Apogee (m) | 375.7074352 | 1232.64 |
| Speed at Inertial Apogee (m/sec) | 22.97667008 | 75.3828 |
| Throw Distance at Inertial Apogee (m) | 197.0616243 | 646.528 |
| Inertial Ascent Time (sec) | 9.83 | |
| | | |
| Baro Altitude at Burnout (m) | 198.5512127 | 651.415 |
| Inertial Altitude at Burnout (m) | 189.0612923 | 620.28 |
| Velocity at Burnout (m/sec) | 69.28726536 | 227.32 |
| | | |
| Maximum Velocity (m/sec) | 75.8360046 | 248.806 |
| Maximum Mach Number | 0.219505055 | |
| | | |
| Max Acceleration (m/sec^2) | 45.32425833 | 148.702 |
| Min Acceleration (m/sec^2) | -13.0231687 | -42.727 |
| | | |
| Burning Time (sec) | 3.73 | |
| Impulse (ns) | 103.0340416 | 458.162 |
| | | |
| Specific Impuse (sec) | 195.2890014 | |
| Effective Exhaust Velocity (m/sec) | 1868.561984 | 6130.45 |



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# VULCANITE $90^0$

| | |
|---|---|
| TEMPERATURE C | 13.333 |
| LAUNCH SITE ALTITUDE (m) | 0 |
| | |
| DIAMETER (cm) | 5.74 |
| LAUNCH MASS (g) | 779 |
| PROPELLANT MASS (g) | 47.9 |
| LAUNCH ROD LENGTH (cm) | 134.62 |
| LAUNCH ANGLE (Degrees) | 90 |

| | | English |
|---|---|---|
| RSquare | 0.998350955 | |
| Barometric Apogee (m) | 345.1587638 | 1132.47 |
| Inertial Alt at Barometric Apogee (m) | 337.0996746 | 1106.02 |
| Throw Distance at Barometric Apogee (m) | 1.07493E-12 | 3.5E-12 |
| Barometric Ascent Time (sec) | 8.34 | |
| | | |
| Inertial Apogee (m) | 337.1158987 | 1106.08 |
| Speed at Inertial Apogee (m/sec) | 0.001002782 | 0.00329 |
| Throw Distance at Inertial Apogee (m) | 1.08562E-12 | 3.6E-12 |
| Inertial Ascent Time (sec) | 8.28 | |
| | | |
| Baro Altitude at Burnout (m) | 85.15330344 | 279.388 |
| Inertial Altitude at Burnout (m) | 75.1449472 | 246.551 |
| Velocity at Burnout (m/sec) | 82.64271511 | 271.151 |
| | | |
| Maximum Velocity (m/sec) | 84.62358377 | 277.65 |
| Maximum Mach Number | 0.249466569 | |
| | | |
| Max Acceleration (m/sec^2) | 414.2570253 | 1359.18 |
| Min Acceleration (m/sec^2) | -18.5990495 | -61.023 |
| | | |
| Burning Time (sec) | 1.395 | |
| Impulse (ns) | 76.13941015 | 338.82 |
| | | |
| Specific Impuse (sec) | 162.0889164 | |
| Effective Exhaust Velocity (m/sec) | 1589.549272 | 5215.31 |



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# STRETCH INITIATOR $75^0$

| | |
|---|---|
| TEMPERATURE C | 20.55555556 |
| LAUNCH SITE ALTITUDE (m) | 396 |
| | |
| DIAMETER (cm) | 6.7 |
| LAUNCH MASS (g) | 923 |
| PROPELLANT MASS (g) | 53.8 |
| LAUNCH ROD LENGTH (cm) | 134.62 |
| LAUNCH ANGLE (Degrees) | 75 |

| | | English |
|---|---|---|
| RSquare | 0.978796297 | |
| Barometric Apogee (m) | 275.6744909 | 904.444 |
| Inertial Alt at Barometric Apogee (m) | 277.366782 | 909.996 |
| Throw Distance at Barometric Apogee (m) | 176.9595093 | 580.576 |
| Barometric Ascent Time (sec) | 8.01 | |
| | | |
| Inertial Apogee (m) | 277.3753698 | 910.024 |
| Speed at Inertial Apogee (m/sec) | 20.56231512 | 67.4617 |
| Throw Distance at Inertial Apogee (m) | 176.1376781 | 577.88 |
| Inertial Ascent Time (sec) | 7.97 | |
| | | |
| Baro Altitude at Burnout (m) | 140.6096873 | 461.318 |
| Inertial Altitude at Burnout (m) | 114.9132008 | 377.012 |
| Velocity at Burnout (m/sec) | 70.03631061 | 229.778 |
| | | |
| Maximum Velocity (m/sec) | 74.7848372 | 245.357 |
| Maximum Mach Number | 0.217791868 | |
| | | |
| Max Acceleration (m/sec^2) | 71.51152357 | 234.618 |
| Min Acceleration (m/sec^2) | -16.0321307 | -52.599 |
| | | |
| Burning Time (sec) | 2.53 | |
| Impulse (ns) | 91.14419619 | 405.592 |
| | | |
| Specific Impuse (sec) | 172.7531869 | |
| Effective Exhaust Velocity (m/sec) | 1694.130041 | 5558.17 |

# STRETCH INITIATOR $80^0$

| | |
|---|---|
| TEMPERATURE C | 20.55555556 |
| LAUNCH SITE ALTITUDE (m) | 396 |
| | |
| DIAMETER (cm) | 6.7 |
| LAUNCH MASS (g) | 920 |
| PROPELLANT MASS (g) | 53.8 |
| LAUNCH ROD LENGTH (cm) | 134.62 |
| LAUNCH ANGLE (Degrees) | 80 |

| | | English |
|---|---|---|
| RSquare | 0.973094801 | |
| Barometric Apogee (m) | 315.8341261 | 1036.2 |
| Inertial Alt at Barometric Apogee (m) | 332.3751432 | 1090.47 |
| Throw Distance at Barometric Apogee (m) | 125.4430021 | 411.558 |
| Barometric Ascent Time (sec) | 7.79 | |
| | | |
| Inertial Apogee (m) | 336.9851566 | 1105.59 |
| Speed at Inertial Apogee (m/sec) | 17.39052244 | 57.0555 |
| Throw Distance at Inertial Apogee (m) | 135.3155456 | 443.949 |
| Inertial Ascent Time (sec) | 8.38 | |
| | | |
| Baro Altitude at Burnout (m) | 148.4989151 | 487.201 |
| Inertial Altitude at Burnout (m) | 120.1025325 | 394.037 |
| Velocity at Burnout (m/sec) | 75.69735349 | 248.351 |
| | | |
| Maximum Velocity (m/sec) | 79.41719189 | 260.555 |
| Maximum Mach Number | 0.231301051 | |
| | | |
| Max Acceleration (m/sec^2) | 58.15524918 | 190.798 |
| Min Acceleration (m/sec^2) | -14.7299741 | -48.327 |
| | | |
| Burning Time (sec) | 2.42 | |
| Impulse (ns) | 94.64107298 | 421.153 |
| | | |
| Specific Impuse (sec) | 179.3811088 | |
| Effective Exhaust Velocity (m/sec) | 1759.127751 | 5771.42 |



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# STRETCH INITIATOR $90^0$

| | | |
|---|---|---|
| TEMPERATURE C | 24.44444444 | |
| LAUNCH SITE ALTITUDE (m) | 396 | |
| | | |
| DIAMETER (cm) | 6.7 | |
| LAUNCH MASS (g) | 922 | |
| PROPELLANT MASS (g) | 53.8 | |
| LAUNCH ROD LENGTH (cm) | 134.62 | |
| LAUNCH ANGLE (Degrees) | 90 | |
| | | English |
| **RSquare** | 0.980938191 | |
| **Barometric Apogee (m)** | 295.580531 | 969.752 |
| **Inertial Alt at Barometric Apogee (m)** | 306.921491 | 1006.96 |
| **Throw Distance at Barometric Apogee (m)** | 1.03901E-12 | 3.4E-12 |
| **Barometric Ascent Time (sec)** | 7.28 | |
| | | |
| **Inertial Apogee (m)** | 313.0488111 | 1027.06 |
| **Speed at Inertial Apogee (m/sec)** | 0.02351675 | 0.07715 |
| **Throw Distance at Inertial Apogee (m)** | 1.17783E-12 | 3.9E-12 |
| **Inertial Ascent Time (sec)** | 8.36 | |
| | | |
| **Baro Altitude at Burnout (m)** | 166.4628581 | 546.138 |
| **Inertial Altitude at Burnout (m)** | 153.3044774 | 502.967 |
| **Velocity at Burnout (m/sec)** | 63.84286675 | 209.458 |
| | | |
| **Maximum Velocity (m/sec)** | 72.40786675 | 237.559 |
| **Maximum Mach Number** | 0.209505061 | |
| | | |
| **Max Acceleration (m/sec^2)** | 54.81 | 179.823 |
| **Min Acceleration (m/sec^2)** | -15.4 | -50.525 |
| | | |
| **Burning Time (sec)** | 3.02 | |
| **Impulse (ns)** | 94.42955751 | 420.212 |
| | | |
| **Specific Impuse (sec)** | 178.9802059 | |
| **Effective Exhaust Velocity (m/sec)** | 1755.196236 | 5758.52 |



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# WEASEL $70^0$ (First Launch at $70^0$)

| | |
|---|---|
| TEMPERATURE C | 21.11111111 |
| LAUNCH SITE ALTITUDE (m) | 253 |
| | |
| DIAMETER (cm) | 4.1148 |
| LAUNCH MASS (g) | 565 |
| PROPELLANT MASS (g) | 36 |
| LAUNCH ROD LENGTH (cm) | 69 |
| LAUNCH ANGLE (Degrees) | 70 |

| | | English |
|---|---|---|
| RSquare | 0.987628415 | |
| Barometric Apogee (m) | 364.801661 | 1196.86 |
| Inertial Alt at Barometric Apogee (m) | 352.3190657 | 1155.9 |
| Distance at Barometric Apogee (m) | 242.408261 | 795.303 |
| Barometric Ascent Time (sec) | 6.405 | |
| | | |
| Inertial Apogee (m) | 367.4707777 | 1205.61 |
| Speed at Inertial Apogee (m/sec) | 43.38932096 | 142.353 |
| Throw Distance at Inertial Apogee (m) | 264.1023445 | 866.478 |
| Inertial Ascent Time (sec) | 6.985 | |
| | | |
| Baro Altitude at Burnout (m) | 145.0491246 | 475.883 |
| Inertial Altitude at Burnout (m) | 168.0092169 | 551.211 |
| Velocity at Burnout (m/sec) | 90.54386604 | 297.06 |
| | | |
| Maximum Velocity (m/sec) | 93.73775791 | 307.539 |
| Maximum Mach Number | 0.272879345 | |
| | | |
| Max Acceleration (m/sec^2) | 79.46931806 | 260.726 |
| Min Acceleration (m/sec^2) | -19.093785 | -62.644 |
| | | |
| Burning Time (sec) | 2.85 | |
| Impulse (ns) | 70.15901095 | 312.208 |
| | | |
| Specific Impuse (sec) | 198.7285582 | |
| Effective Exhaust Velocity (m/sec) | 1948.861415 | 6393.9 |



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# WEASEL 70$^0$ (Second Launch at 70$^0$)

| | |
|---|---|
| TEMPERATURE C | 11.11111111 |
| LAUNCH SITE ALTITUDE | 253 |
| | |
| DIAMETER | 4.1148 |
| LAUNCH MASS (g) | 566 |
| PROPELLANT MASS (g) | 36 |
| LAUNCH ROD LENGTH (cm) | 69 |
| LAUNCH ANGLE | 70 |

| | | English |
|---|---|---|
| RSquare | 0.990863995 | |
| Barometric Apogee (m) | 388.787505 | 1275.55 |
| Inertial Alt at Barometric Apogee (m) | 379.5084073 | 1245.11 |
| Throw Distance at Barometric Apogee (m) | 296.8347694 | 973.867 |
| Barometric Ascent Time (sec) | 8.27 | |
| | | |
| Inertial Apogee (m) | 381.3660603 | 1251.2 |
| Speed at Inertial Apogee (m/sec) | 30.39042591 | 99.7061 |
| Throw Distance at Inertial Apogee (m) | 315.6986018 | 1035.76 |
| Inertial Ascent Time (sec) | 8.88 | |
| | | |
| Baro Altitude at Burnout (m) | 184.9221312 | 606.7 |
| Inertial Altitude at Burnout (m) | 162.9085672 | 534.477 |
| Velocity at Burnout (m/sec) | 90.19946539 | 295.93 |
| | | |
| Maximum Velocity (m/sec) | 95.10559189 | 312.026 |
| Maximum Mach Number | 0.281628079 | |
| | | |
| Max Acceleration (m/sec^2) | 618.486803 | 2029.16 |
| Min Acceleration (m/sec^2) | -20.0716207 | -65.852 |
| | | |
| Burning Time (sec) | 2.665 | |
| Impulse (ns) | 69.8131432 | 310.668 |
| | | |
| Specific Impuse (sec) | 197.7488722 | |
| Effective Exhaust Velocity (m/sec) | 1939.253978 | 6362.38 |



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# WEASEL $80^0$

| | | |
|---|---|---|
| TEMPERATURE C | 27.22222222 | |
| LAUNCH SITE ALTITUDE m | 253 | |
| | | |
| DIAMETER (cm) | 4.1148 | |
| LAUNCH MASS (g) | 558 | |
| PROPELLANT MASS (g) | 36 | |
| LAUNCH ROD LENGTH (cm) | 76.2 | |
| LAUNCH ANGLE | 80 | |
| | | |
| DELTA T | 0.01 | |
| | | English |
| RSquare | 0.994452382 | |
| Barometric Apogee (m) | 534.0506573 | 1752.13 |
| Inertial Alt at Barometric Apogee (m) | 552.7653905 | 1813.53 |
| Throw Distance at Barometric Apogee (m) | 211.8272851 | 694.971 |
| Barometric Ascent Time (sec) | 10.02 | |
| | | |
| Inertial Apogee (m) | 554.7465708 | 1820.03 |
| Speed at Inertial Apogee (m/sec) | 22.98707571 | 75.4169 |
| Throw Distance at Inertial Apogee (m) | 216.304407 | 709.66 |
| Inertial Ascent Time (sec) | 10.23 | |
| | | |
| Baro Altitude at Burnout (m) | 209.5256521 | 687.42 |
| Inertial Altitude at Burnout (m) | 202.7679548 | 665.249 |
| Velocity at Burnout (m/sec) | 100.2613381 | 328.941 |
| | | |
| Maximum Velocity (m/sec) | 104.3381627 | 342.317 |
| Maximum Mach Number | 0.300691662 | |
| | | |
| Max Acceleration (m/sec^2) | 75.88726493 | 248.974 |
| Min Acceleration (m/sec^2) | -15.1850522 | -49.82 |
| | | |
| Burning Time (sec) | 2.9 | |
| Impulse (ns) | 75.20223808 | 334.65 |
| | | |
| Specific Impuse (sec) | 213.0137262 | |
| Effective Exhaust Velocity (m/sec) | 2088.951058 | 6853.51 |



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## WEASEL $90^0$

| | |
|---|---|
| TEMPERATURE | 25.55555556 |
| LAUNCH SITE ALTITUDE | 253 |
| | |
| DIAMETER | 4.1148 |
| LAUNCH MASS | 561 |
| PROPELLANT MASS | 36 |
| LAUNCH ROD LENGTH | 76.2 |
| LAUNCH ANGLE | 90 |
| | |
| DELTA T | 0.01 |

| | | English |
|---|---|---|
| RSquare | 0.990641712 | |
| Barometric Apogee (m) | 480.1369154 | 1575.25 |
| Inertial Alt at Barometric Apogee (m) | 494.8596614 | 1623.56 |
| Throw Distance at Barometric Apogee (m) | 1.71575E-12 | 5.6E-12 |
| Barometric Ascent Time (sec) | 9.84 | |
| | | |
| Inertial Apogee (m) | 495.4814198 | 1625.6 |
| Speed at Inertial Apogee (m/sec) | 0.04771675 | 0.15655 |
| Throw Distance at Inertial Apogee (m) | 1.77169E-12 | 5.8E-12 |
| Inertial Ascent Time (sec) | 10.19 | |
| | | |
| Baro Altitude at Burnout (m) | 167.7270827 | 550.286 |
| Inertial Altitude at Burnout (m) | 167.6595214 | 550.064 |
| Velocity at Burnout (m/sec) | 95.23416675 | 312.448 |
| | | |
| Maximum Velocity (m/sec) | 99.56956675 | 326.672 |
| Maximum Mach Number | 0.287655136 | |
| | | |
| Max Acceleration (m/sec^2) | 541.8533 | 1777.73 |
| Min Acceleration (m/sec^2) | -17.24 | -56.562 |
| | | |
| Burning Time (sec) | 2.51 | |
| Impulse (ns) | 71.3443595 | 317.482 |
| | | |
| Specific Impuse (sec) | 202.0861113 | |
| Effective Exhaust Velocity (m/sec) | 1981.787764 | 6501.93 |



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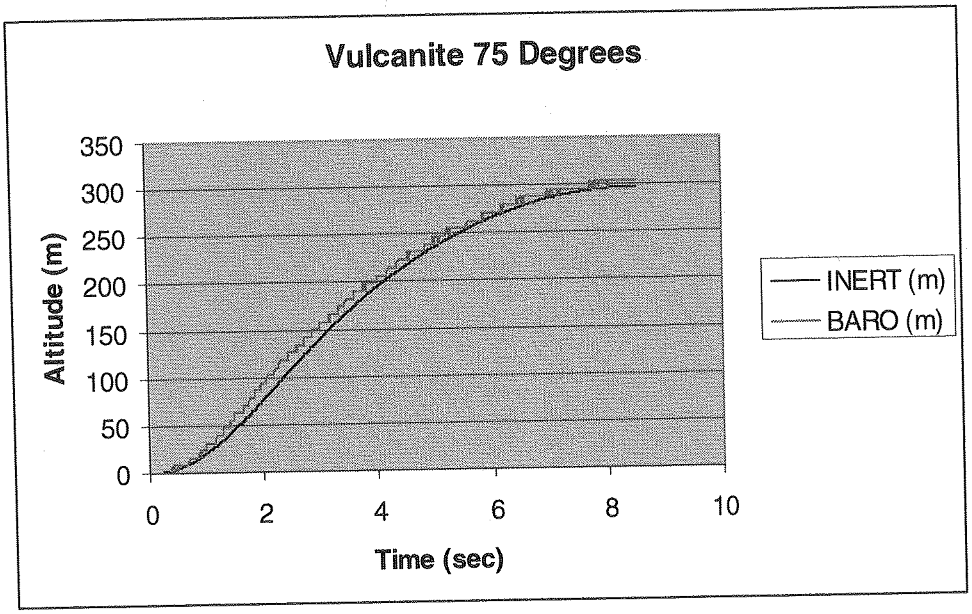


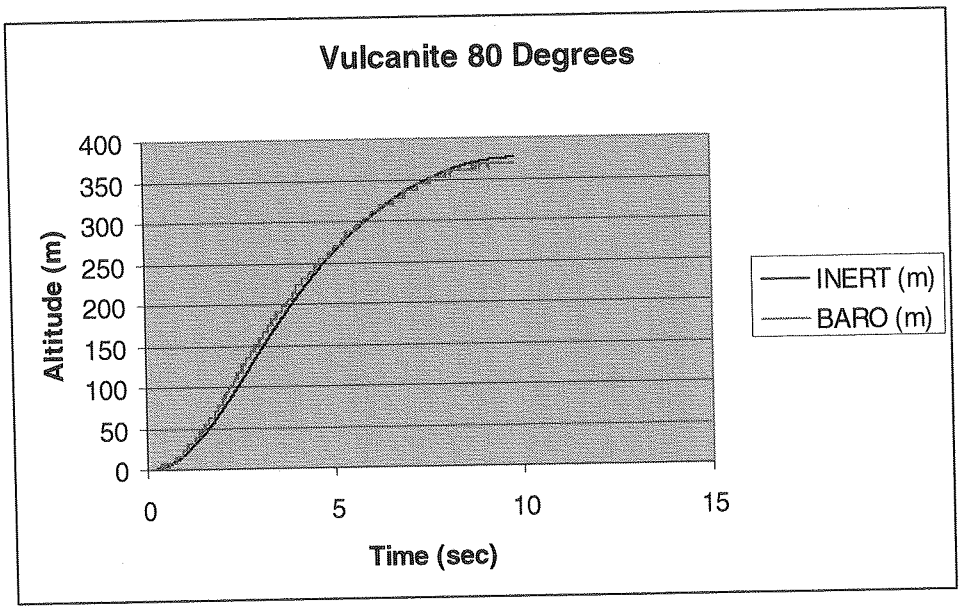




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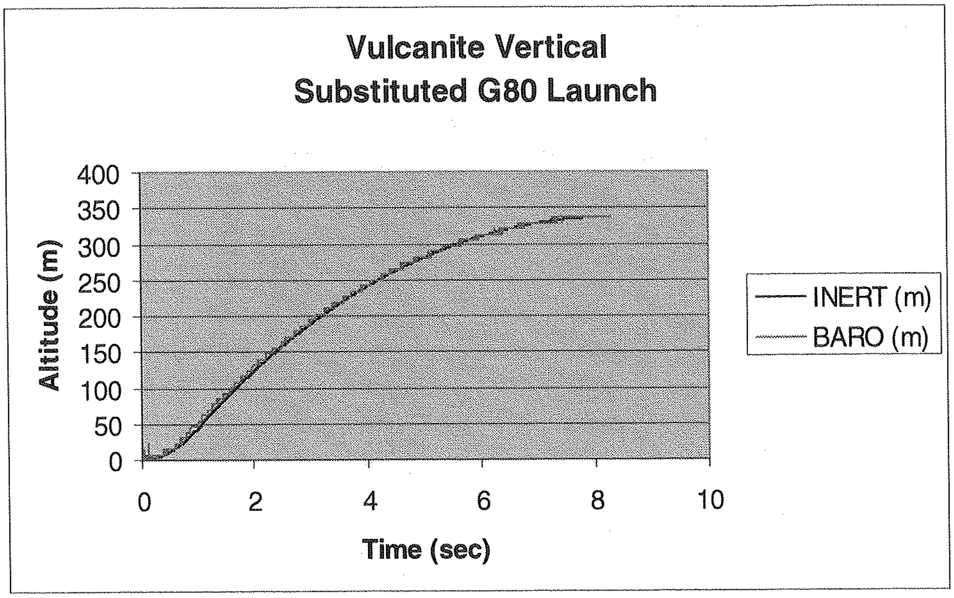


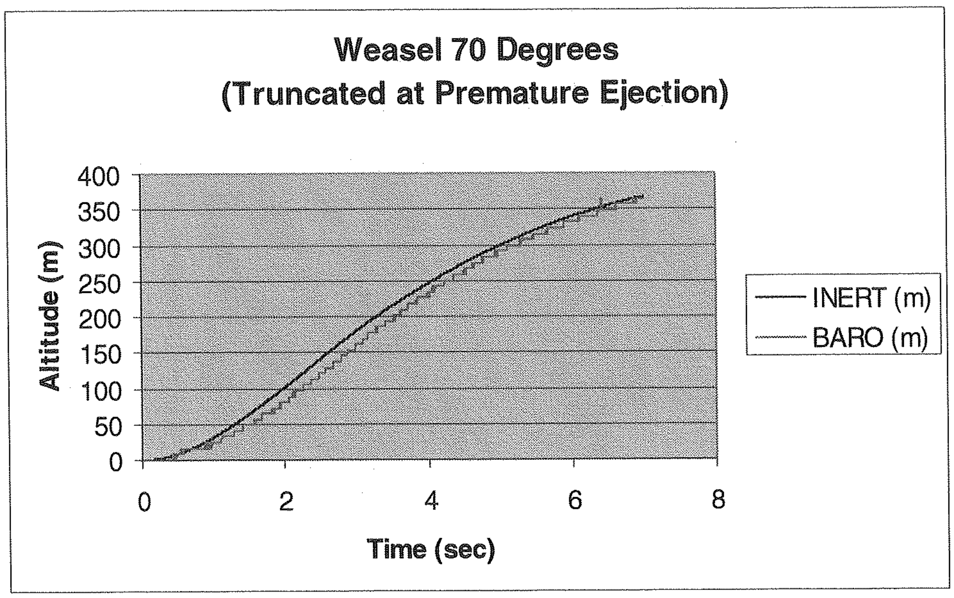




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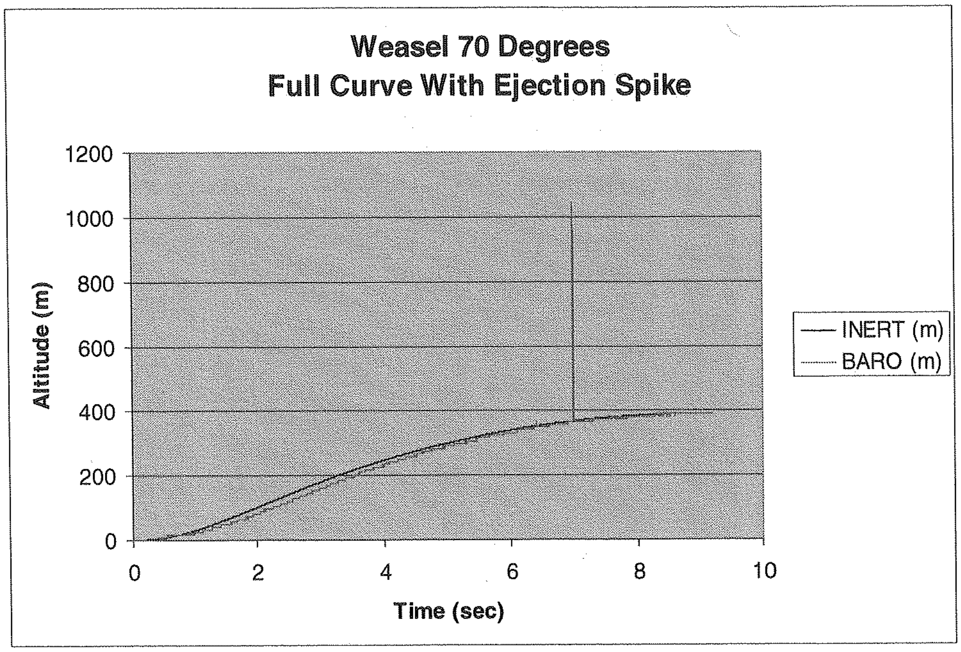


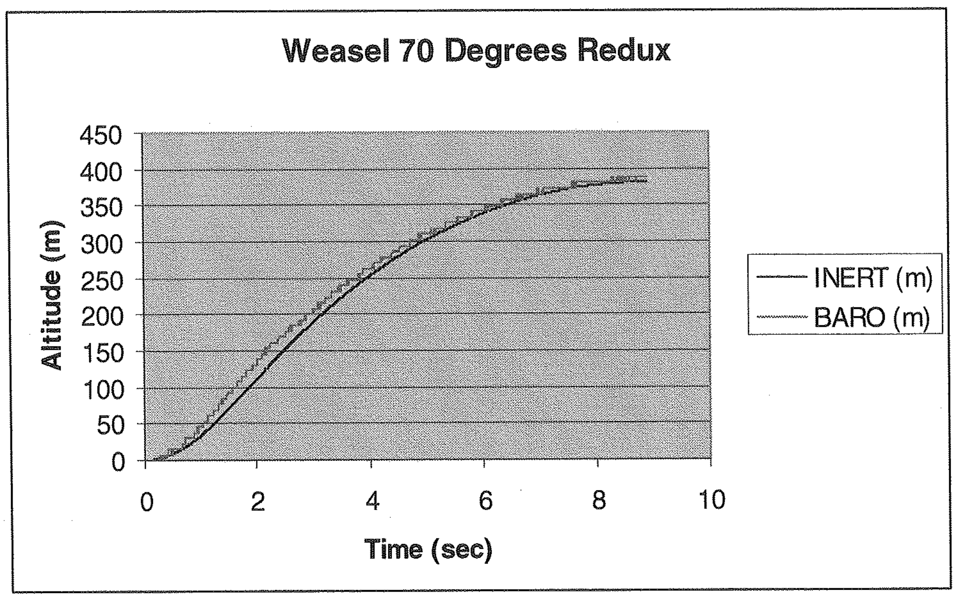




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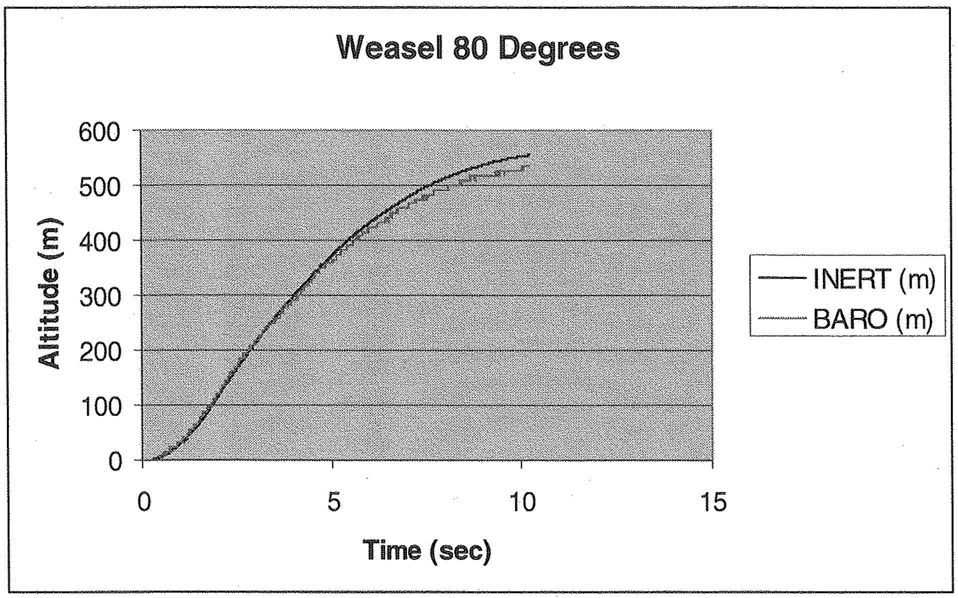


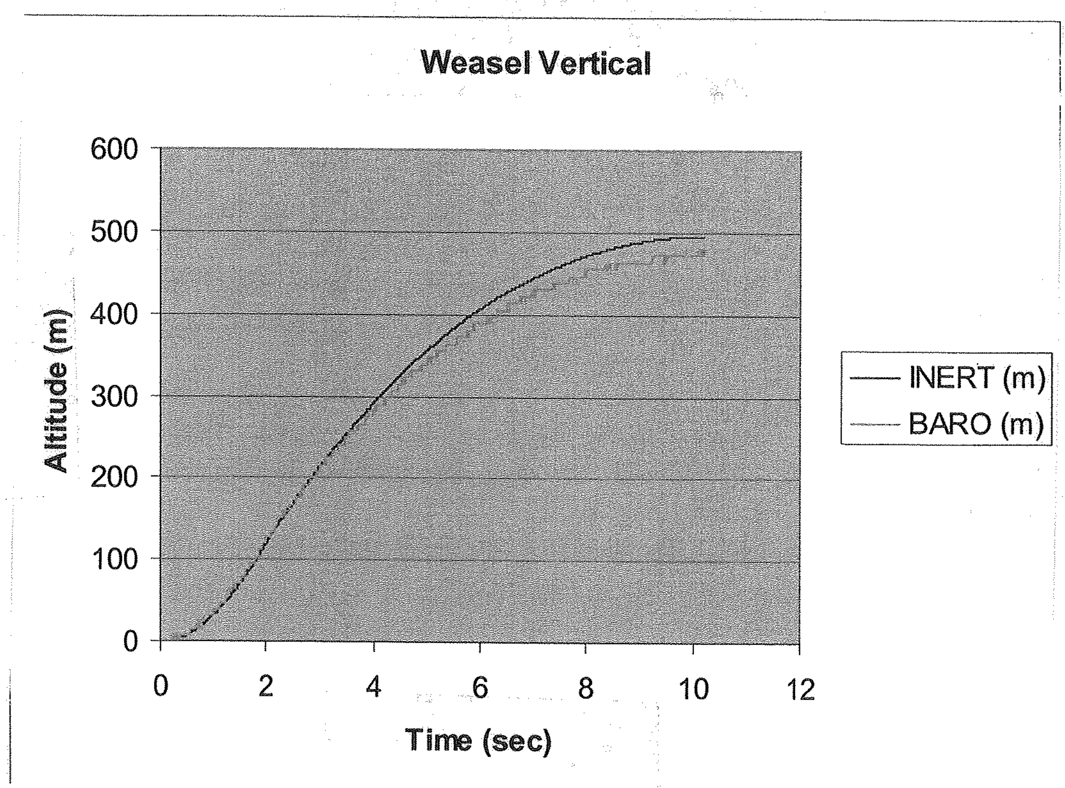




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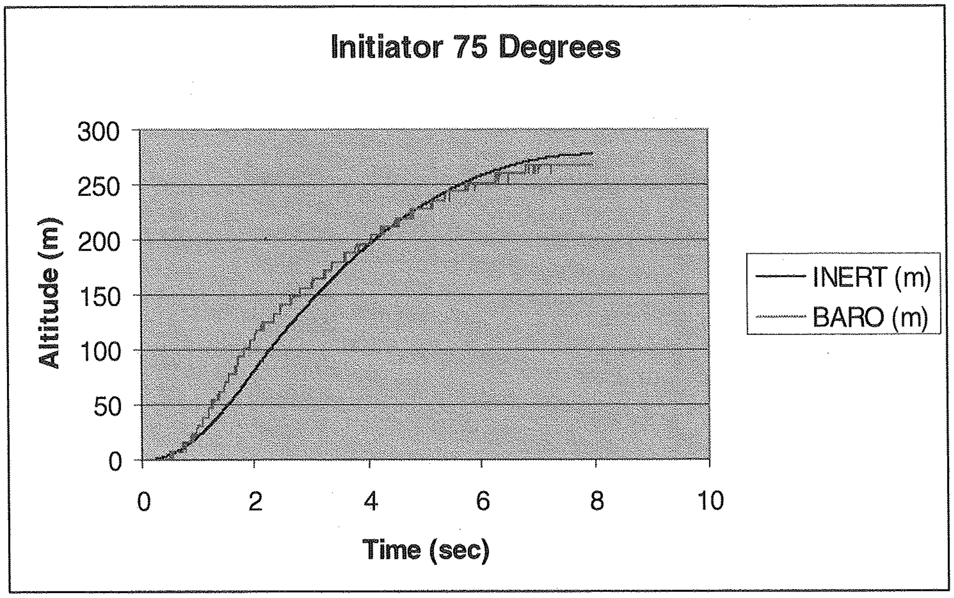


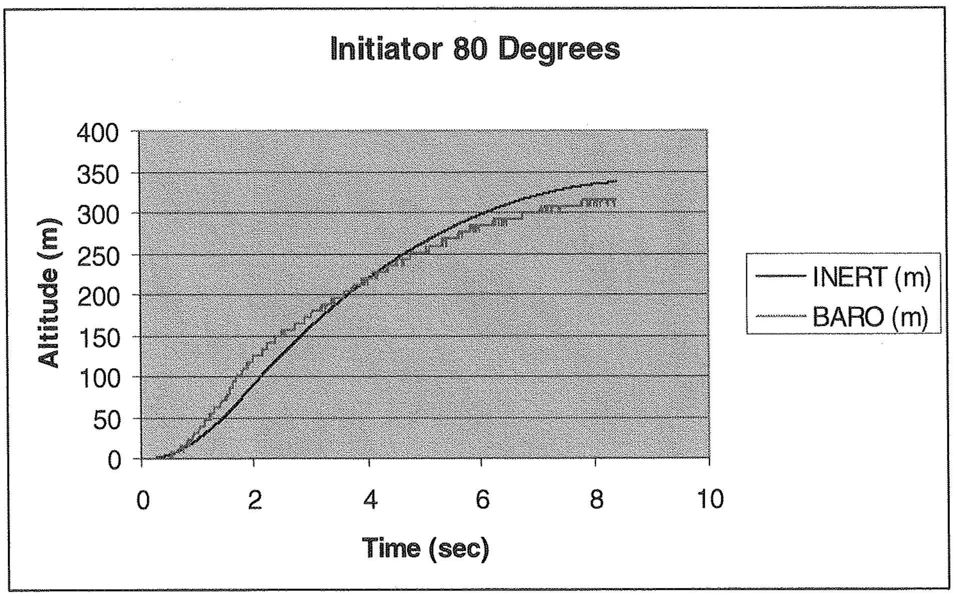




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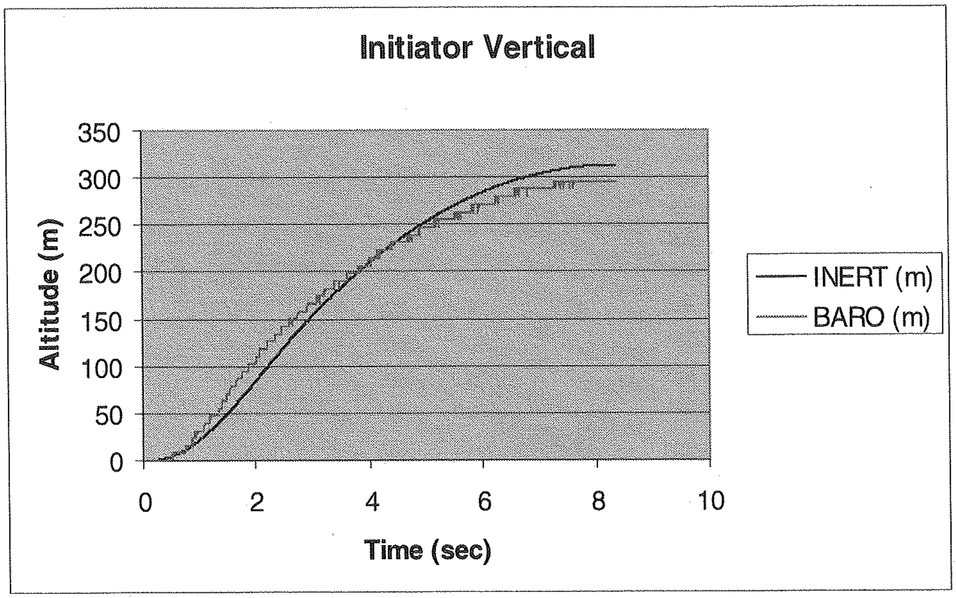




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# APPENDIX C
# ERRORS IN ALTIMETER DATA

**EDITORIAL NOTE:** This material was originally in the main body of this paper. It was removed in order to spare the NARAM-49 R&D judges the task of reading through more paper than was necessary. Then, during the presentation one judge, Jerry King, asked about nonstandard temperature lapse, the coverage of which had been edited out. Most of the original chapter now appears here in this appendix. What is not here remains in the main body.

Whereas accelerometer data tend to be regarded with healthy suspicion, altimeter data are typically accepted at face value. Divergences between barometric altitudes and inertial altitudes are too frequently dismissed, without second thought, as bad accelerometer data. Altimeter errors do exist, and they arise from many sources. A few are listed below:

1) Data held to low precision;
2) Poor onboard approximations;
3) Small number of values in baseline;
4) Launch detect errors;
5) Ejection spikes and outliers
6) Altimeter delay and other aerodynamic effects;
7) Nonstandard temperature lapse
8) Failure to correct for ambient temperature;
9) Departures of the atmosphere from ideality;
10) Fundamental limitations to the technology.

The biggest culprit in all of these is item 8, failure to correct for ambient temperature. A somewhat less important, but equally conspicuous source of error is item 6, altimeter delay. We now examine each source in some detail.

**LOW PRECISION**
Item 1 should be obvious. If an altimeter is holding only 8 bits of pressure information, it can hold only 256 distinct values. Ambient pressure consumes a significant portion of that range. Data held to low precision accounts for the typical stair stepping pattern frequently observed in altimeter data. Effects from this source are examined in *Kidwell* (reference 4). This source of error can afflict accelerometer data as well, though it is more prevalent in altimeters when the instruments are packaged together.

**POOR ONBOARD APPROXIMATIONS**
Because onboard electronics must be compact, it may be impractical to use the full mathematical model, which is presented again below.

$$Altitude = \left(\frac{BaseTemperature}{LapseRate}\right) * \left\{1 - \left[\frac{Pressure}{BasePressure}\right]^{\frac{LapseRate}{HC}}\right\} + BaseAltitude$$



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Here, *BaseAltitude* is the altitude at which *BasePressure* and *BaseTemperature* are measured. For most rocket applications, we are interested in altitude above launch site ground level, so *BasAltitude* would be taken as zero. (At least one instrument, the *ALTACC*, is calibrated from sea level.)

*HC is the hydrostatic constant.* $HC \equiv .03418155$ Kelvins per meter.
*LapseRate* is the rate of temperature decline, which is normally assumed to be 6.5 Kelvins per meter in the troposphere.

Instead of this formula, instruments tend to use approximations – sometimes linear ones. Instruments that allow their data to be downloaded to computers can analyze data with full models (See Reference 8). These data may vary somewhat from the more approximate models used to beep out the maximum altitude. Strangely, some of the models used by computer software employ polynomial approximations, even though the full model is available. Although the values tend to be very good, they can impart characteristic patterns in error curves.

**SMALL NUMBERS OF VALUES IN AVERAGE BASE PRESSURE**
The above formula involves a value for base pressure. Therefore, the accuracy of every number in the dataset is dependent upon the accuracy of this value. Typically, altimeters use the average of a certain number of pressure values before launch detect to represent base pressure. The number tends to be between 3 and 400. The standard error of the average value goes down with the square root of the number of values, so 400 is a whole lot better than 3. Since noise in a denominator propagates badly, the stability of the base pressure value is vital to accuracy.

**LAUNCH DETECT ERRORS**
Given that baseline values are taken on the launch pad, it is important to know what numbers were actually taken on the launch pad. Errors in launch detect can wreck subsequent data, particularly when base pressure is computed from a small number of readings. Launch detection is important in inertial data streams as well.

Some instruments (e.g.; the RDAS) have optional electronic switch mechanisms to detect launches. These employ cords attached to the launch pad or to parachutes. These can be made to work well, but excessive slack in the cord or a misadventure can defeat the intent.

In general, it is important to inspect the raw baseline data to make certain that they represent a true baseline.

**EJECTION SPIKES AND OUTLIERS**
Ejection tends to shock the altimeter and it produces a spike. Some instruments are designed to ignore such values. Others report them as maximum altitude. The difference is typically on the order of 300 feet. In addition, altimeter data are typically quite noisy,



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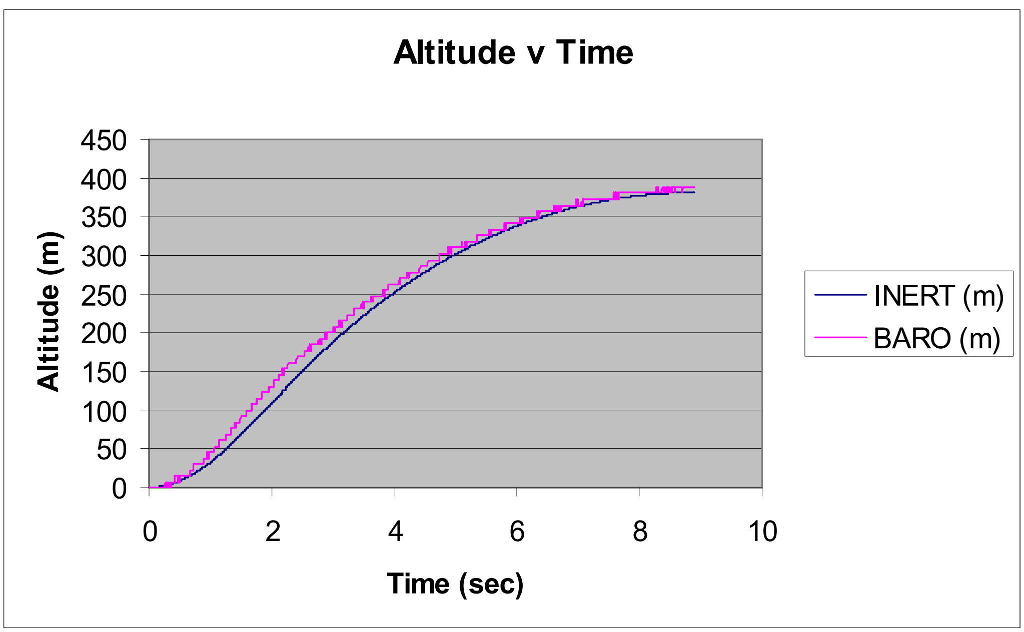
so even when the ejections spike is removed, maximum altitude may be assessed as a value that is obviously inaccurate.

**ALTIMETER DELAY AND OTHER AERODYNAMIC EFFECTS**
Altimeter data are analyzed on the assumption that air pressure in the altimeter bay accurately reflects ambient air pressure outside the rocket. This assumption may fail because of static port size, placement, or design. For example:

1) The static port may be so small that the internal temperature takes too long to adjust at high speeds (*Altimeter Delay*);
2) The static port may be too large, and turbulence at high speeds affects the readings;
3) The static port is in a position where pressure, at high speeds, tends to be lower or higher than ambient pressure;
4) The static port is in the downwash of an obstruction or is rough around the edges, and the resulting turbulence affects altimeter readings.

Graphs are presented in the main body of this paper.

Although altimeter delay is well known and understood in hobby rocketry, the opposite problem also occurs with great frequency. One of the results of the Bernoulli equations is that pressure declines as speed increases. Thus, air rushing by a static port may be sampled at lower than ambient pressure. The symptoms again present in the region of highest velocity, but barometric altitude appears to rise faster than inertial altitude.




Notice that the errors in the graph above go pretty far up the curve. This is not a characteristic of the Bernoulli effect; it is because the flight was at $70^0$, which is



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substantially off-vertical. There is considerable speed all the way to apogee, and along with the speed goes aerodynamic effects.

Note that there are also areas of many rockets that sample air at higher than ambient pressure.

**NONSTANDARD TEMPERATURE LAPSE**

The standard tropospheric temperature lapse rate is 6.5 Kelvins/Celsius per kilometer of altitude. Actual lapse rates tend to be greater in arid areas, and smaller in humid areas.

In July of 2005, I had occasion to take my own soundings while on a plane from Beijing to Newark. The flight was equipped with a GPS position, altitude and temperature display. The standard value corresponds well with my own measured lapse rate of 6.38 degrees per km over Beijing, but the lapse rate over Newark was only 5.78 degrees/km when I recorded it.

How important are such errors? Here is an evaluation of errors that would have arisen over Newark from this source

| **Reading** | **Assumed Lapse** | **Actual Lapse** | **Real Alt** | **Error** | **%Error** |
|---|---|---|---|---|---|
| 999.63ft | 6.5 Deg/km | 5.78 Deg/km | 1000ft | .375ft | .04% |
| 5000 ft | 6.5 Deg/km | 5.78 Deg/km | 4990.56 | 9.44ft | .19% |

<u>Evidently, the atmospheric model is robust to small variations in lapse rate.</u> At this writing, NOAA provides a web site with actual soundings over various geographic regions. (Reference 6)

**FAILURE TO CORRECT FOR AMBIENT TEMPERATURE**

This topic is covered in some detail in the main body of this paper. It is mentioned here for completeness.

**DEPARTURES FROM IDEALITY**

Altimeter data analysis is based on hydrostatic equilibrium, the first law of thermodynamics, and the ideal gas law. Departures from any of these laws undermines altimeter analysis to some extent; departures from the ideal gas law, $PV = NRT$, are included.

The primary way that the atmosphere may violate this law is in phase transitions, which take place, notably, in clouds. It is not uncommon to observe anomalies in altitude/time curves as a rocket of balloon passes through a cloud bank. The effect is discussed further in the next section.

**LIMITATIONS TO THE TECHNOLOGY**

As John Demar once pointed out to me, barometric altimeters are designed to keep planes apart when they are in the same geographic region at the same time. It is something of a stretch to compare readings at different locations and different times. Indeed, a close examination of the principles underlying the barometric altimeter is cause for at least as much suspicion as is given to accelerometers.

The barometric model is grounded in three principles: hydrostatic equilibrium, the equation of state for ideal gasses, and the first law of thermodynamics. Of these, the first two are suspect.

Hydrostatic equilibrium is based on the assumption of still air. Under that assumption, the weight of a column of air above a given altitude is balanced by the pressure of the air below the given altitude. The still air assumption doesn't really hold in the Troposphere. Indeed, the very name derives from the Greeks word, *Tropos*, meaning *To Turn*. Thus, reliance on this principle is something of a leap.

Reliance on the ideal gas law is a leap too. Whereas dry air conforms to it very well, humid air departs somewhat from the model, because ideal gasses do not change state. Cloud banks are clear evidence that components of real air do change state. As a result, it is difficult to derive the observed average temperature lapse rate of 6.5 Kelvins per kilometer, much less demonstrate that it should be constant. The ideal gas law yields a predicted lapse rate for dry air that is indeed constant, but it comes to about 9.74 Kelvins per kilometer. Observed values above very arid regions (like the Black Rock Desert) don't get much higher than 8 Kelvins per kilometer.

The tropospheric altimeter formula is the result of substituting an observed average lapse rate into a framework based on assumptions that incorrectly predicted the theoretical lapse rate in the first place.

Given all of the above, it is surprising that altimeters work as well as they do.



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# APPENDIX E
# UNBIASED NOISE

**EDITORIAL NOTE:** This is yet another insertion of text that was originally in the main body of the paper.

Accelerometers are not perfect. Along with meaningful data, they register a (hopefully small) component from a host of irrelevant influences, which manifests as noise. If we were to average these influences over the data stream, the figure would tend to zero with increasing numbers of samples. We call such a component, *Unbiased Noise*.

How can we assume that the noise term is unbiased? In fact, there are biased noise components, which are discussed in the body of this paper. When we remove them from the noise term, we will be left with unbiased noise.

It is widely believed that unbiased noise cancels out with integration, because it averages to zero. That is not actually the case. The pertinent average, in this case, is the sum of the noise values divided by the number of values. For this ratio to tend to zero in large samples, it is sufficient for the denominator to increase much faster than the numerator. It is not necessary for the numerator itself, the summed noise, to go to zero.

Consider a coin flipping contest between two persons. In a large number of flips, the ratio of each person's winnings to total flips tends to 0.5; however the absolute value of the difference between the two contestants' wins tends to grow large. We could reframe the example as a drunkard's walk from bar to men's room. With each step, the drunkard stumbles a random step above or below the bee line. To expect that these vertical errors cancel out is to expect that the drunk stands a better chance of finding the men's room if he starts very much farther away: an absurd conclusion.



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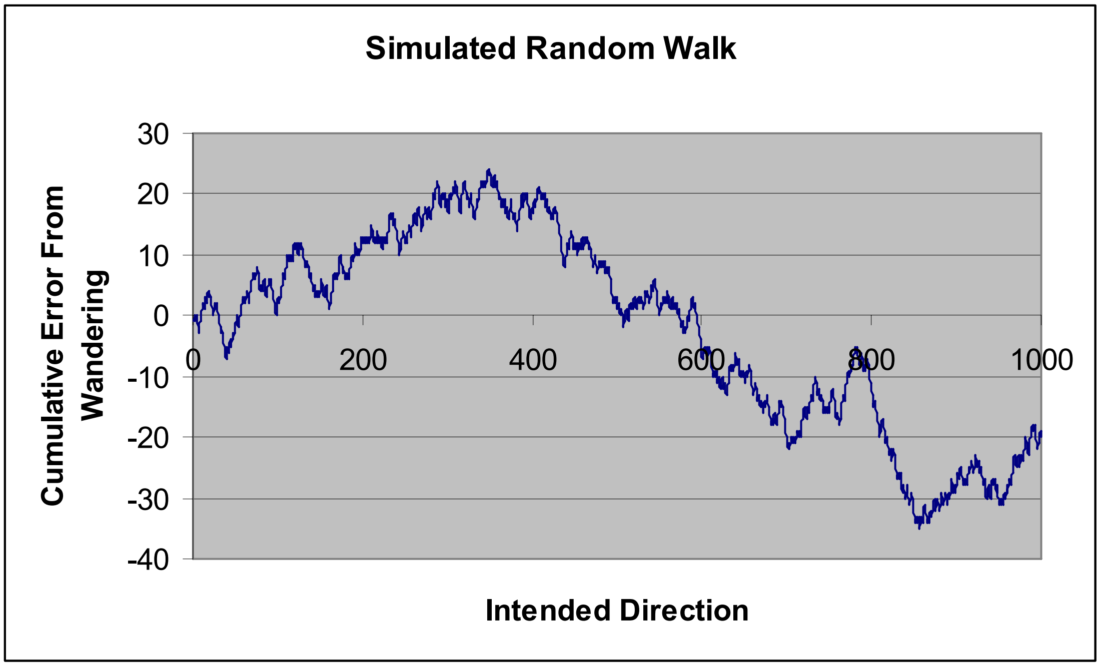


Analogously, the cumulative error from integrating a sequence of noisy accelerometer readings tends to become large, even as the percentage error in velocity tends to zero. That is because numerical integrals are weighted sums. In the context of noise, cumulative error is called *drift*. The expected value of drift is computable; the direction of drift is not. Over HPR flights, the total amount of drift from unbiased noise is usually small compared with everything else. Over the flight of a cruise missile or an ICBM, on the other hand, such drift can be considerable. The technology for minimizing drift is involved, and much is classified. Commercial inertial navigation systems resynchronize with satellites from time to time.

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